1. We start at the origin, which has coordinates \((0, 0, 0)\). First we move 4 units along the positive \(x\)-axis, affecting only the \(x\)-coordinate, bringing us to the point \((4, 0, 0)\). We then move 3 units straight downward, in the negative \(z\)-direction. Thus only the \(z\)-coordinate is affected, and we arrive at \((4, 0, -3)\).

3. The distance from a point to the \(yz\)-plane is the absolute value of the \(x\)-coordinate of the point. \(C(2, 4, 6)\) has the \(x\)-coordinate with the smallest absolute value, so \(C\) is the point closest to the \(yz\)-plane. \(A(-4, 0, -1)\) must lie in the \(xz\)-plane since the distance from \(A\) to the \(xz\)-plane, given by the \(y\)-coordinate of \(A\), is 0.

4. The projection of \((2, 3, 5)\) onto the \(xy\)-plane is \((2, 3, 0)\), onto the \(yz\)-plane, \((0, 3, 5)\); onto the \(xz\)-plane, \((2, 0, 5)\).

The length of the diagonal of the box is the distance between the origin and \((2, 3, 5)\), given by 
\[
\sqrt{(2 - 0)^2 + (3 - 0)^2 + (5 - 0)^2} = \sqrt{38} \approx 6.16
\]

7. (a) We can find the lengths of the sides of the triangle by using the distance formula between pairs of vertices:
\[
|PQ| = \sqrt{(7 - 3)^2 + (0 - (-2))^2 + (1 - (-3))^2} = \sqrt{16 + 4 + 16} = 6
\]
\[
|QR| = \sqrt{(1 - 7)^2 + (2 - 0)^2 + (1 - 1)^2} = \sqrt{36 + 4 + 0} = \sqrt{40} = 2 \sqrt{10}
\]
\[
|RP| = \sqrt{(3 - 1)^2 + (-2 - 2)^2 + (-3 - 1)^2} = \sqrt{4 + 16 + 16} = 6
\]

The longest side is \(QR\), but the Pythagorean Theorem is not satisfied. \(|PQ|^2 + |RP|^2 \neq |QR|^2\). Thus \(PQR\) is not a right triangle. \(PQR\) is isosceles, as two sides have the same length.

(b) Compute the lengths of the sides of the triangle by using the distance formula between pairs of vertices:
\[
|PQ| = \sqrt{(4 - 2)^2 + (1 - (-1))^2 + (1 - 0)^2} = \sqrt{4 + 4 + 1} = 3
\]
\[
|QR| = \sqrt{(4 - 4)^2 + (-5 - 1)^2 + (4 - 4)^2} = \sqrt{36 + 9} = \sqrt{45} = 3 \sqrt{5}
\]
\[
|RP| = \sqrt{(2 - 4)^2 + (-1 - (-5))^2 + (0 - 4)^2} = \sqrt{4 + 16 + 16} = 6
\]

Since the Pythagorean Theorem is satisfied by \(|PQ|^2 + |RP|^2 = |QR|^2\), \(PQR\) is a right triangle. \(PQR\) is not isosceles, as no two sides have the same length.
9. (a) First we find the distances between points:

\[ |AB| = \sqrt{(3 - 2)^2 + (7 - 4)^2 + (-2 - 2)^2} = \sqrt{26} \]
\[ |BC| = \sqrt{(1 - 3)^2 + (3 - 7)^2 + (3 - (-2))^2} = \sqrt{45} = 3\sqrt{5} \]
\[ |AC| = \sqrt{(1 - 2)^2 + (3 - 4)^2 + (3 - 2)^2} = \sqrt{3} \]

In order for the points to lie on a straight line, the sum of the two shortest distances must be equal to the longest distance. Since \(\sqrt{26} + \sqrt{3} \neq 3\sqrt{5}\), the three points do not lie on a straight line.

(b) First we find the distances between points:

\[ |DE| = \sqrt{(1 - 0)^2 + (-2 - (-5))^2 + (4 - 5)^2} = \sqrt{11} \]
\[ |EF| = \sqrt{(3 - 1)^2 + (4 - (-2))^2 + (2 - 4)^2} = \sqrt{44} = 2\sqrt{11} \]
\[ |DF| = \sqrt{(3 - 0)^2 + (4 - (-5))^2 + (2 - 5)^2} = \sqrt{99} = 3\sqrt{11} \]

Since \(|DE| + |EF| = |DF|\), the three points lie on a straight line.

19. (a) Since the sphere touches the \(xy\)-plane, its radius is the distance from its center, \((2, -3, 6)\), to the \(xy\)-plane, namely 6.

Therefore \(r = 6\) and an equation of the sphere is \((x - 2)^2 + (y + 3)^2 + (z - 6)^2 = 6^2 = 36\).

(b) The radius of this sphere is the distance from its center \((2, -3, 6)\) to the \(yz\)-plane, which is 2. Therefore, an equation is \((x - 2)^2 + (y + 3)^2 + (z - 6)^2 = 4\).

(c) Here the radius is the distance from the center \((2, -3, 6)\) to the \(xz\)-plane, which is 3. Therefore, an equation is \((x - 2)^2 + (y + 3)^2 + (z - 6)^2 = 9\).

35. We need to find a set of points \(P(x, y, z) \mid |AP| = |BP| \).

\[ \sqrt{(x + 1)^2 + (y - 5)^2 + (z - 3)^2} = \sqrt{(x - 6)^2 + (y - 2)^2 + (z + 2)^2} \quad \Rightarrow \]
\[ (x + 1)^2 + (y - 5)^2 + (z - 3)^2 = (x - 6)^2 + (y - 2)^2 + (z + 2)^2 \quad \Rightarrow \]
\[ x^2 + 2x + 1 + y^2 - 10y + 25 + z^2 - 6z + 9 = x^2 - 12x + 36 + y^2 - 4y + 4 + z^2 + 4z + 4 \quad \Rightarrow \]
\[ 14x - 6y - 10z = 9 \]

Thus the set of points is a plane perpendicular to the line segment joining \(A\) and \(B\) (since this plane must contain the perpendicular bisector of the line segment \(AB\)).

13. \(a + b = (5 + (-3), -12 + (-6)) = (2, -18)\)

\(2a + 3b = (10, -24) + (-9, -18) = (1, -42)\)

\(|a| = \sqrt{5^2 + (-12)^2} = \sqrt{169} = 13\)

\(|a - b| = |(5 - (-3), -12 - (-6))| = |(8, -6)| = \sqrt{8^2 + (-6)^2} = \sqrt{100} = 10\)
16. \( \mathbf{a} + \mathbf{b} = (2 \mathbf{i} - 4 \mathbf{j} + 4 \mathbf{k}) + (2 \mathbf{j} - \mathbf{k}) = 2 \mathbf{i} - 2 \mathbf{j} + 3 \mathbf{k} \)

\[ 2\mathbf{a} + 3\mathbf{b} = 2(2 \mathbf{i} - 4 \mathbf{j} + 4 \mathbf{k}) + 3(2 \mathbf{j} - \mathbf{k}) = 4 \mathbf{i} - 8 \mathbf{j} + 8 \mathbf{k} + 6 \mathbf{j} - 3 \mathbf{k} = 4 \mathbf{i} - 2 \mathbf{j} + 5 \mathbf{k} \]

\[ |\mathbf{a}| = \sqrt{2^2 + (-4)^2 + 4^2} = \sqrt{36} = 6 \]

\[ |\mathbf{a} - \mathbf{b}| = |(2 \mathbf{i} - 4 \mathbf{j} + 4 \mathbf{k}) - (2 \mathbf{j} - \mathbf{k})| = |2 \mathbf{i} - 6 \mathbf{j} + 5 \mathbf{k}| = \sqrt{2^2 + (-6)^2 + 5^2} = \sqrt{65} \]

22. From the figure, we see that the horizontal component of the force \( \mathbf{F} \) is \( |\mathbf{F}| \cos 38^\circ = 50 \cos 38^\circ \approx 39.4 \text{ N} \), and the vertical component is \( |\mathbf{F}| \sin 38^\circ = 50 \sin 38^\circ \approx 30.8 \text{ N} \).

\[ \text{F} \]

\[ 38^\circ \]

28. Call the two tensile forces \( \mathbf{T}_3 \) and \( \mathbf{T}_5 \), corresponding to the ropes of length 3 m and 5 m. In terms of vertical and horizontal components,

\[ \mathbf{T}_3 = -|\mathbf{T}_3| \cos 52^\circ \mathbf{i} + |\mathbf{T}_3| \sin 52^\circ \mathbf{j} \quad (1) \quad \text{and} \quad \mathbf{T}_5 = |\mathbf{T}_5| \cos 40^\circ \mathbf{i} + |\mathbf{T}_5| \sin 40^\circ \mathbf{j} \quad (2) \]

The resultant of these forces, \( \mathbf{T}_3 + \mathbf{T}_5 \), counterbalances the force of gravity acting on the decoration [which is \( -59 \mathbf{j} \approx -5(9.8) \mathbf{j} = -49 \mathbf{j} \)]. So \( \mathbf{T}_3 + \mathbf{T}_5 = 49 \mathbf{j} \). Hence

\[ \mathbf{T}_3 + \mathbf{T}_5 = (-|\mathbf{T}_3| \cos 52^\circ + |\mathbf{T}_5| \cos 40^\circ) \mathbf{i} + (|\mathbf{T}_3| \sin 52^\circ + |\mathbf{T}_5| \sin 40^\circ) \mathbf{j} = 49 \mathbf{j} \]

Thus \( -|\mathbf{T}_3| \cos 52^\circ + |\mathbf{T}_5| \cos 40^\circ = 0 \) and \( |\mathbf{T}_3| \sin 52^\circ + |\mathbf{T}_5| \sin 40^\circ = 49 \).

From the first of these two equations \( |\mathbf{T}_3| = |\mathbf{T}_5| \frac{\cos 40^\circ}{\cos 52^\circ} \). Substituting this into the second equation gives

\[ |\mathbf{T}_3| = \frac{49}{\cos 40^\circ \tan 52^\circ + \sin 40^\circ} \approx 30 \text{ N}. \]

Therefore, \( |\mathbf{T}_3| = |\mathbf{T}_5| \frac{\cos 40^\circ}{\cos 52^\circ} \approx 38 \text{ N}. \) Finally, from (1) and (2),

\[ \mathbf{T}_3 \approx -23 \mathbf{i} + 30 \mathbf{j}, \text{ and } \mathbf{T}_5 \approx 23 \mathbf{i} + 19 \mathbf{j}. \]

33. (orthogonal) \( \mathbf{a} \cdot \mathbf{b} = (\mathbf{b} \cdot \text{proj}_\mathbf{a} \mathbf{b}) \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{a} - \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \mathbf{a} \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{a} - \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} |\mathbf{a}|^2 = \mathbf{b} \cdot \mathbf{a} - \mathbf{a} \cdot \mathbf{b} = 0. \)

So they are orthogonal by (7).

43. For convenience, consider the unit cube positioned so that its back left corner is at the origin, and its edges lie along the coordinate axes. The diagonal of the cube that begins at the origin and ends at \( (1, 1, 1) \) has vector representation \( (1, 1, 1) \).

The angle \( \theta \) between this vector and the vector of the edge which also begins at the origin and runs along the \( x \)-axis [that is, \( (1, 0, 0) \)] is given by

\[ \cos \theta = \frac{(1, 1, 1) \cdot (1, 0, 0)}{|(1, 1, 1)|| (1, 0, 0)|} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1} \left( \frac{1}{\sqrt{3}} \right) \approx 55^\circ. \]
48. Let the figure be called quadrilateral $ABCD$. The diagonals can be represented by $\overrightarrow{AC}$ and $\overrightarrow{BD}$. $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ and $\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{BC} - \overrightarrow{DC} = \overrightarrow{BC} - \overrightarrow{AB}$ (Since opposite sides of the object are of the same length and parallel, $\overrightarrow{AB} = \overrightarrow{DC}$.) Thus

$$\overrightarrow{AC} \cdot \overrightarrow{BD} = (\overrightarrow{AB} + \overrightarrow{BC}) \cdot (\overrightarrow{BC} - \overrightarrow{AB}) = \overrightarrow{AB} \cdot (\overrightarrow{BC} - \overrightarrow{AB}) + \overrightarrow{BC} \cdot (\overrightarrow{BC} - \overrightarrow{AB})$$

$$= \overrightarrow{AB} \cdot \overrightarrow{BC} - |\overrightarrow{AB}|^2 + |\overrightarrow{BC}|^2 - \overrightarrow{AB} \cdot \overrightarrow{BC} = |\overrightarrow{BC}|^2 - |\overrightarrow{AB}|^2$$

But $|\overrightarrow{AB}|^2 = |\overrightarrow{BC}|^2$ because all sides of the quadrilateral are equal in length. Therefore $\overrightarrow{AC} \cdot \overrightarrow{BD} = 0$, and since both of these vectors are nonzero this tells us that the diagonals of the quadrilateral are perpendicular.

50. (a) The Triangle Inequality states that the length of the longest side of a triangle is less than or equal to the sum of the lengths of the two shortest sides.

(b) $|a + b|^2 = (a + b) \cdot (a + b) = (a \cdot a) + 2(a \cdot b) + (b \cdot b) = |a|^2 + 2(a \cdot b) + |b|^2$

$$\leq |a|^2 + 2|a||b| + |b|^2 \quad \text{[by the Cauchy-Schwartz Inequality]}$$

$$= (|a| + |b|)^2$$

Thus, taking the square root of both sides, $|a + b| \leq |a| + |b|$. 