1. Work the following problems from the text:
   (a) Section 11.4: 19, 28, 30, 31, 34
   (b) Section 11.5: 20, 32, 35, 47
   (c) Section 11.6: 23, 26, 40, 41, 44

2. Parabolic mirrors have the nice focusing property that all rays coming in parallel to their axis of symmetry are focused to the same point. Suppose that we have a parabolic mirror surface that is implicitly defined by the equation $F(x, y, z) = 0$, where
   \[ F(x, y, z) = x^2 + y^2 - z. \]
   (a) Use the gradient of $F(x, y, z)$ to find a vector normal to the surface at the point $(1, 2, 5)$.
   (b) What is the standard equation for the plane tangent, $T$, to our paraboloid at $(1, 2, 5)$? What is the normal vector to $T$?
   (c) Consider a point on a surface with a local normal $n$, and a ray of light that reflects off the surface at that point. Let the direction of the incoming and reflected rays be $w_{in}$ and $w_{out}$, respectively. The formula for finding the direction of the reflected ray, given the incoming ray and the local normal, $w_{out} = w_{in} - 2\left(\frac{w_{in} \cdot n}{n \cdot n}\right)n$. Draw a picture and explain why this gives us vectors where the angle of incidence equals the angle of reflection.
   (d) Consider an incoming ray that is parallel to the $z$-axis so that a vector in the direction of this ray is $w_{in} = -k$. What is $w_{out}$ if $w_{in}$ intersects the mirror at $(1, 2, 5)$?
   (e) What is the parameterization for the line, $L$, that intersects the surface at the point $(1, 2, 5)$ and is in the direction of $w_{out}$?
   (f) The point where $L$ intersects the $z$-axis is the focal point of the parabolic mirror. Determine the coordinates of the focal point for our mirror.

3. There’s a nasty virus going around, and a school district has hired you to determine if they should cancel school. They’ve decided that if a child’s probability of becoming infected is above 10%, the school will have to close until the probability is below that threshold. Your friend Bob is a microbiologist and he’s determined that any given child’s probability of becoming infected is given by $P(t, n, c) = \frac{tn}{c}$, where $t$ is the time (hours) the students are at school, $n$ is the number of infected students who come to school, and $c$ is the total number of students who come to school. You know the time the students are at school is known with absolute certainty, but it varies among the different schools. The number of students who come to school averages to about 5,000 for this district, with about a 10% fluctuation. Bob estimates that about 100 of these students will be contagious, with a 5% error in that approximation.
   (a) Based on the average number of students who come to school, and Bob’s estimate for the number of these students that will be infected, determine the approximate probability that students will be infected (this will depend on $t$).
   (b) Determine an upper bound for the percent error in this approximation.
(c) What is the probability for the worst case scenario (i.e. the upper bound you determined in part (b) is equal to the error)?

(d) What is the cutoff time? In other words, a school open for how many hours, \( t \) or longer, will have to close?

4. Suppose, at a given point \( P \), the directional derivatives of \( f(x, y) \) are known in two nonparallel directions given by unit vectors \( \hat{u} \) and \( \hat{v} \). In particular, suppose \( \frac{df}{ds} = A \) in the \( \hat{u} \) direction, and \( \frac{df}{ds} = B \) in the \( \hat{v} \) direction. Is it possible to find \( \nabla f \) at the point \( P \)? If so, how do you do it?