Due date: Friday, Nov. 7 2014, before 10 AM

Note: updated Monday 11/3/14 to reflect that on problem 3 (c) (i), you do not need to fully justify your answer.

Theme: compact operators

Instructions: Problems marked with “Collaboration Allowed” mean that collaboration with your fellow students is OK and in fact recommended, although direct copying is not allowed. The internet is allowed for basic tasks (e.g., looking up definitions on wikipedia) but it is not permissible to search for proofs or to post requests for help on forums such as http://math.stackexchange.com/. On these problems, please write down the names of the students that you worked with.

On problems marked “No Collaboration,” collaboration with anyone is forbidden. Internet usage is forbidden, but using the course text is allowed, as well as any book mentioned on the syllabus. These problems can be viewed as take-home exams.

An arbitrary subset of these questions will be graded.

Problem 1: Problems 5.10 and 5.11 from the book. No Collaboration

Problem 2: Problem 5.9 from the book. Collaboration Allowed

Problem 3: Collaboration Allowed. Note: you are allowed to use ANY resources on this problem (e.g., other books, the internet), though you may not post in an online forum asking for direct help. In the following, provide examples and give a quick justification of why your example is valid.

(a) Give an example of $T \in B(X,Y)$ where $X$ is finite dimensional and $Y$ is infinite dimensional.
(b) Give an example of $T \in B(X,Y)$ where $X$ is infinite dimensional and $Y$ is finite dimensional. (This example should be different than the one used in part (d)).
(c) Let $T \in B(X,Y)$. Which of the following necessarily imply that $T$ is a compact operator? (and if not, justify why not)
   i. $X$ is finite dimensional (Note: you do not need to justify this one. It is a bit tricky)
   ii. $Y$ is finite dimensional
   iii. Both $X$ and $Y$ are finite dimensional
(d) Theorem 5.35 says that if $T : X \to Y$ is linear, then $T$ is bounded if $X$ is finite dimensional. Give an example where $Y$, but not $X$, is finite dimensional, and $T$ is unbounded.

Problem 4: Collaboration Allowed. Review of other student groups’ prelim solutions.

(a) Instructions for the review. In the same groups as last week, you will review another groups’ prelim solutions. Use the other groups’ \LaTeX file, and add two new sections. The first section should include a summary of the main theorems in their proof and whether you think there are any critical flaws (and if so, what are the fixes?). The second
section should be a final version of the proof, e.g., copy their original proof and make any modifications that are beneficial. If their proof is very nice, you may not need to modify it all. Or it may be that their proof is correct but you can make it shorter, more elegant, or add more understanding, so please do so.

(b) To determine which group to review, take the question that is the first one numbered above the problem that you did last week (not all problems were solved, so it may not be the number directly above yours). Cycle back to the beginning if necessary. All the \LaTeX files are collected in folders with names like Problem_04 at http://amath.colorado.edu/faculty/becker/prelimSolutions/.

(c) When you are finished, email the \LaTeX source files and pdf output to the instructor.