3. Assignment 3
Due Wednesday, February 11

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(1) Prove that
(a) if all singular values of matrix $A \in C^{n \times n}$ are equal, then $A = \gamma U$, where $U$ is a unitary matrix and $\gamma$ is a constant
(b) if $A \in C^{n \times n}$ is nonsingular and $\lambda$ is an eigenvalue of $A$, then $\|A^{-1}\|^{-1} \leq |\lambda| \leq \|A\|$
(c) Show that any square matrix $A \in C^{n \times n}$ may be represented in the form $A = SU$, where $S$ is a hermitian nonnegative definite matrix and $U$ is a unitary matrix. Show that if $A$ is invertible such representation is unique.

(2) Atkinson, Problem 6 page 649. (Prove a perturbation estimate for the eigenvector corresponding to a simple eigenvalue on page 600. Read a hint on page 649/650).