In this problem set you will be exploring different methods for solving the following parabolic PDE:

\[
\begin{align*}
    u_t - u_{xx} &= 2 & 0 \leq x \leq 1, \quad t > 0 \\
    u(0, t) &= u(1, t) = 0 \\
    u(x, 0) &= \sin(\pi x) + x (1 - x)
\end{align*}
\]

the exact solution of which is given by 

\[ u(x, t) = e^{-\pi^2 t} \sin(\pi x) + x (1 - x) \]

1. Approximate the solution to the PDE at time \( t = 0.25 \) using \( h = 0.1 \) and \( k = 0.01 \) for each of the following methods. For each method, make a table displaying the solution value and errors at each of the interior grid points.

   (a) Forward Euler in time, centered differences in space
   (b) Backward Euler in time, centered differences in space
   (c) Crank-Nicolson in time, centered differences in space

2. Clearly describe a set of numerical experiments you could use to verify the accuracy of each method in Problem 1 (e.g. Forward Euler should be first-order accurate in time and second-order accurate in space). Carry out your experiments and discuss your results.

3. (a) Show analytically that the Crank-Nicolson scheme for the homogeneous parabolic model problem is second-order accurate in both time and space.
   (b) Use Von Neumann analysis to show that the Crank-Nicolson scheme for the homogeneous model problem is unconditionally stable.