This assignment is to help you prepare for the second midterm. It is not to be turned in.

1. Consider the following elliptic partial differential equation

\[-u_{xx} - u_{yy} = 2 \text{ in } \Omega = [0,1]^2\]
\[u = 1 \text{ on } \partial \Omega\]

(a) Write down the difference scheme for the given problem using centered finite differences in space.
(b) Write down (explicitly) the matrix equation that results from discretizing the PDE on $[0,1]^2$ where the mesh is broken into $n = 4$ subintervals in each spatial direction.
(c) Discuss some numerical techniques you might use to solve the resulting linear system.

2. Consider the following wave equation

\[u_{tt} - 4u_{xx} = 0 \text{ for } 0 < x < 1\]
\[u(0,t) = u(1,t) = 0\]
\[u(x,0) = \sin(\pi x)\]
\[u_t(x,0) = \cos(\pi x/2)\]

(a) Write down the explicit difference scheme that uses centered differences in both time and space.
(b) Show that the given difference scheme is second-order accurate in both time and space.
(c) The explicit difference scheme requires using an alternative method for computing the approximation at the first time-step. Explain how you could do this for the given problem without harming the second-order accuracy of the method.
(d) Use Von Neumann analysis to derive a stability condition on $h$, and $k$ for the explicit method.

3. Consider the following advection equation

\[u_t - 2u_x = 0 \text{ for } 0 < x < 10\]
\[u(0,t) = u(10,t)\]
\[u(x,0) = e^{-(x-5)^2}\]

(a) Use the method of characteristics to find an exact solution to the given advection equation.
(b) Write down a difference scheme for the advection equation using Explicit Euler in time and upwinded differences in space.
(c) Use Von Neumann analysis to derive a stability condition on $h$ and $k$ for the given explicit upwinded scheme.