1. Generate a 7-entry small table of $\Gamma(x)$ for $x = 3.11, 3.12, \ldots, 3.17$.

   a. Use Stirling’s interpolation formula to approximate $\Gamma(\pi)$ from the tabulated values, and compare with the correct value.

   b. Use inverse interpolation (also based on Stirling’s formula) to find the $x$-value for which $\Gamma(x) = 2.3$.

   (Matlab is suitable for all parts of this problem)

2. Suppose that we are given three points $x_0, x_1, x_2$ with $x_0 \neq x_1$, and values for $p(x_0), p(x_1), p'(x_2), p''(x_2)$. Determine if this information uniquely specifies a cubic polynomial $p(x)$.

3. The error in Hermite interpolation is given by

   $$f(x) - H_{2n}(x) = \frac{f^{(2n)}(\xi)}{(2n)!} \psi(x)^2$$

   where $\psi(x) = (x-x_1)(x-x_2)\cdots(x-x_n)$. The proof that is given in Atkinson (pages 160-161) uses a limit argument of points pairwise coinciding, and properties of the expressions $f[x_1, x_2, \ldots, x_n]$ from Newton’s interpolation formula. Maybe this is ‘elegant’, but it is definitely not very straightforward. We can in fact proceed almost identically to how we did for regular polynomial interpolation. Thus, choose some arbitrary location $t$ and verify that (1) holds at this location. For this purpose, consider the function

   $$G(x) = f(x) - H_{2n}(x) - R \psi(x)^2,$$

   etc. Complete this argument to arrive at (1).

4. The cubic Hermite interpolation polynomial based on data for $f(a), f'(a), f(b), f'(b)$ is obtained by the Lagrange approach in equation (3.6.12) in the text book. Derive the same polynomial by instead using the Newton interpolation procedure (in its enhanced version that also allows the use of derivative information). Verify that the two procedures give the identical result.