This assignment is due under your TA’s office door by **4:00pm on Friday**, October 2nd. Remember that you are encouraged to discuss the problems with your classmates, but all work turned in must be your own.

1. Work the following problems from the text:

   (a) Section 11.3: 6, 52, 56, 64 b and d, 68, 73

2. Consider the function \( f(x, y) = \frac{\sin(\pi(x^2 + y^2))}{x^2 + y^2} \).

   (a) What are the domain and range of this function?

   (b) Does a limit of \( f(x, y) \) exist at the origin? If so, what is the value?

   (c) Is \( f(x, y) \) continuous? If not, why?

   (d) What is the linearization, \( L(x, y) \), of \( f(x, y) \) around the point \((1, 1)\)?

3. Assume we have a function \( f(x, y, z) = c \), where \( c \) is a constant, that relates the three variables, \( x, y, \) and \( z \). Shortly you will find that

   \[
   \left( \frac{\partial x}{\partial y} \right)_z \left( \frac{\partial y}{\partial z} \right)_x \left( \frac{\partial z}{\partial x} \right)_y = -1 \tag{1}
   \]

   where, for example, \( \left( \frac{\partial x}{\partial y} \right)_z \) means the partial derivative of \( x \) with respect to \( y \) while holding \( z \) constant. Equation (1) is known as the **cyclic relation** and it is frequently used in engineering, physics, and thermodynamics in particular.

   (a) As an example, consider the ideal gas law, \( pv = kT \), where \( p \) is the pressure, \( v \) is the container volume divided by the number of molecules, \( k \) is Boltzmann’s constant, and \( T \) is the temperature in an absolute scale (Rankine or Kelvin). The variables in this expression are \( p, v \) and \( T \). Demonstrate that the cyclic equation holds for the ideal gas law. In other words, show that

   \[
   \left( \frac{\partial p}{\partial v} \right)_T \left( \frac{\partial v}{\partial T} \right)_p \left( \frac{\partial T}{\partial p} \right)_v = -1 \tag{2}
   \]

   Note that all three derivatives in (2) are very reasonable calculations to perform.

   (b) Now, consider the Van der Waals equation of state given by \( \left( p + \frac{a}{v^2} \right)(v - b) = kT \), where \( a \) and \( b \) are constants that account for the intermolecular attractions, and the finite volume of the molecules, respectively. Although equation (2) holds, you will note that the derivative \( \left( \frac{\partial v}{\partial T} \right)_p \) is actually very hard to calculate since it requires one to solve for \( v \) as a function of \( p \) and \( T \), and this requires one to solve a cubic equation. Here is where the real utility of (2) comes forth. If one were to solve for the term \( \left( \frac{\partial v}{\partial T} \right)_p \) from (2), one would find that

   \[
   \left( \frac{\partial v}{\partial T} \right)_p = -\left( \frac{\partial p}{\partial T} \right)_v \left/ \left( \frac{\partial p}{\partial v} \right)_T \right. \tag{3}
   \]

   Each of the partial derivatives on the right–hand side of (3) is relatively easy to calculate, thus allowing one to calculate the left-hand with minimal effort. Determine \( \left( \frac{\partial v}{\partial T} \right)_p \) for Van der Waals’ equation of state in terms of \( p, v \) and \( T \). Be sure to simplify your result.