1. Compute the mean of the $\Gamma(\alpha, \beta)$ distribution by “integrating without integrating”.

2. Suppose that $X$ is uniformly distributed on the interval $(0, 1)$. (We write $X \sim \text{unif}(0, 1)$.)

   (a) Define $Y = -\ln X$. Find the distribution of $Y$. (Name it!)
   
   (b) Without doing any extra work, find the distribution of $Y = -\ln(1 - X)$? Explain.
   
   (c) Find a transformation $y = g(x)$ such that $Y = g(X)$ has the exponential distribution with rate $\lambda$.

3. Suppose that $X_1$ and $X_2$ are independent random variables each with the normal distribution with mean $\mu = 0$ and variance $\sigma^2 = 1$. We write $X_1, X_2 \sim \text{N}(0, 1)$. Find the distribution of $X_1/X_2$. (Name it!)

4. Suppose that $X_1, X_2 \sim \text{N}(0, 1)$.

   Show that $Y_1 := X_1 + X_2$ and $Y_2 := X_1 - X_2$ are independent.

5. Let $X_1, X_2, \ldots, X_n$ be a random sample from the geometric distribution with parameter $p$. Use the geometric distribution that starts from 0. (i.e.: $X_1, X_2, \ldots, X_n \sim \text{geom}_0(p)$).

   (a) Find the distribution of $X_{(1)} := \min(X_1, X_2, \ldots, X_n)$. (Name it!)
   
   (b) Give an interpretation of $X_{(1)}$

6. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the $\text{Beta}(a, 1)$ distribution.

   Find $E[X_{(n)}]$, where $X_{(n)} := \max(X_1, X_2, \ldots, X_n)$.

   $\times$ Derive the moment generating function for the exponential distribution with rate $\lambda$. Be sure to include an explanation as to why we need $t < \lambda$.

8. **Required for 5520 students only:** Suppose that $X_1$ and $X_2$ are independent random variables and that $Y_1 = g_1(X_1)$ and $Y_2 = g_2(X_2)$. Then $Y_1$ and $Y_2$ are independent. Sounds reasonable yes?

   Prove this in the case that $X_1$ and $X_2$ are continuous and $g_1$ and $g_2$ are invertible.

9. **Required for 5520 students only** Let $U_1$ and $U_2$ be independent $\text{unif}(0, 1)$ random variables.

   Show that $X_1$ and $X_2$ defined as
   
   \[
   X_1 = \sqrt{-2 \ln U_1} \cos(2\pi U_2)
   \]
   
   \[
   X_2 = \sqrt{-2 \ln U_1} \sin(2\pi U_2)
   \]

   are independent standard normal random variables.

Problems with an $\times$ through them have been canceled from this assignment. They will appear on the next assignment.