Always justify your answers and no calculators.

1. Consider \( f(x) = x^3 + 2x^2 \) and \( g(x) = 3x^2 - 1 \). Find \( f - g \) and \( 2f/g \) and state their domains.

2. If \( f(x) = \sqrt{2x + 3} \) and \( g(x) = x^2 + 5 \), find \( f \circ g \) and \( f \circ f \) and state their domains.

3. Show that \( \lim_{x \to -4} \frac{1 - x/2}{2} = 3 \) by finding a \( \delta > 0 \) given \( \epsilon > 0 \). Illustrate with a graph.

4. A factory manufactures round balls with volume \( V = 50 \text{ cm}^3 \). Balls are discarded if they do not satisfy the error tolerance of \( \pm 0.4 \text{ cm}^3 \). Shown below are radii \( r \) that produce balls of various sizes.

<table>
<thead>
<tr>
<th>( r \text{ (cm)} )</th>
<th>2.276</th>
<th>2.279</th>
<th>2.282</th>
<th>2.285</th>
<th>2.288</th>
<th>2.292</th>
<th>2.295</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V \text{ (cm}^3) )</td>
<td>49.4</td>
<td>49.6</td>
<td>49.8</td>
<td>50.0</td>
<td>50.2</td>
<td>50.4</td>
<td>50.6</td>
</tr>
</tbody>
</table>

In terms of the formal definition of \( \lim_{x \to a} f(x) = L \), identify \( x, a, f(x), L, \delta, \) and \( \epsilon \).

5. Evaluate the limit, show all work:
   (a) \( \lim_{x \to \infty} \frac{\cos^4(x)}{5 + x^2} \)
   (b) \( \lim_{x \to 4} \frac{x^2 - 4x}{x^2 - 3x - 4} \)
   (c) \( \lim_{h \to 0} \frac{(x + h)^{-1} - x^{-1}}{h} \)
   (d) \( \lim_{t \to 0} \frac{\sin^2(3t)}{t^2} \)
   (e) \( \lim_{x \to -6} \frac{2x + 12}{|x + 6|} \)
   (f) \( \lim_{x \to \infty} \frac{x^5/3 + 1}{x} \)
   (g) \( \lim_{x \to \infty} x^2 - x^4 \)
   (h) \( \lim_{x \to -\infty} \frac{3x + 2}{\sqrt{9x^2 + 1}} \)
   (i) \( \lim_{x \to 1} \frac{2 - x}{x - 1} \)
   (j) \( \lim_{x \to \infty} \frac{x^2 - 2x + 3}{5 - 2x^3} \)

6. If \( f(x) = x^2 + 10 \sin(x) \) show that there is a number \( c \) such that \( f(c) = 1000 \).

7. Find all values of \( a \) and \( b \) that make \( f \) continuous everywhere

\[
 f(x) = \begin{cases} 
 x + 2, & \text{if } x < 0 \\
 2ax^2 + b, & \text{if } 0 \leq x \leq 1 \\
 2 - x, & \text{if } x > 1 
\end{cases} 
\]

Sketch the graph.

8. For the functions given below, either make \( f(x) \) continuous by re-defining it exactly at one point, or explain why \( f(x) \) can’t be made continuous:
   (a) \( f(x) = \frac{x^2 - 2x - 8}{x + 2} \)
   (b) \( f(x) = \frac{x - 7}{|x - 7|} \)
   (c) \( f(x) = \frac{-x^{1/2} + 3}{-x + 9} \)
   (d) \( f(x) = \frac{x - 90}{x + 16x + 60} \)

9. Find all horizontal and vertical asymptotes of \( f(x) = \frac{\sqrt{4x^2 + 1}}{2x - 5} \).

10. The displacement (in meters) of a particle moving in a straight line is given by \( s(t) = t^2 - 8t + 18 \), where \( t \) is measured in seconds.
   (a) Find the average velocity of the particle over the time interval (i) \([3, 4]\) and (ii) \([4, 5]\).
   (b) Find the instantaneous velocity when \( t = 4 \) using the limit definition.

11. Find an equation of the tangent line to the curve \( y = \frac{2x}{(x + 1)^2} \) at the point \( x = 0 \).
12. Use the limit definition of the derivative to find $f'(x)$ given:

(a) $f(x) = \sqrt{2x}$  (b) $f(x) = x^2 - 2x + 1$  (c) $f(x) = \frac{1}{x}$

13. Let $f(x) = \sqrt{x}$.

(a) If $a \neq 0$, find $f'(a)$ using a limit definition of the derivative. Hint: $m^3 - n^3 = (m - n)(m^2 + mn + n^2)$

(b) Show that $y = x^{1/3}$ has a vertical tangent line at $(0, 0)$.

14. Is $f(x) = x^2|x|$ differentiable at $x = 0$? Use the limit definition of the derivative to answer this question.

15. Use the limit definition of the derivative to find $f''(x)$ given $f(x) = x^2$.

16. The cost (in dollars) of producing $x$ units of a certain commodity is $C(x) = 5000 + 10x + 0.05x^2$.

(a) Find the average rate of change of $C$ with respect to $x$ when the production level is changed

i. from $x = 100$ to $x = 105$

ii. from $x = 100$ to $x = 101$

(b) Find the instantaneous rate of change of $C$ with respect to $x$ when $x = 100$.

17. A particle has position function $s(t) = t^3 - 9t^2 + 15t + 10$, where $t$ is measured in seconds and $s$ in feet. Find the total distance traveled in the first 8 seconds.