# Convergence Tests for Series

<table>
<thead>
<tr>
<th>Test Description</th>
<th>Converges</th>
<th>Inconclusive</th>
<th>Diverges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric Series Test (GST): If it converges, then it converges to ( \frac{a}{1-r} ), where ( a ) = first term, ( r = \frac{\text{second}}{\text{first}} )</td>
<td>If (</td>
<td>r</td>
<td>&lt; 1 )</td>
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<tr>
<td>Telescoping Series Test (TST): Must find ( S_n ), use partial fractions if necessary. If it converges then it converges to ( \lim_{n \to \infty} S_n ).</td>
<td>If ( \lim_{n \to \infty} S_n ) exists</td>
<td>Never</td>
<td>If ( \lim_{n \to \infty} S_n ) DNE or is infinite</td>
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<tr>
<td>Nth Term Test for Divergence (NTT)</td>
<td>Never tells you if a series converges</td>
<td>( \lim_{n \to \infty} a_n = 0 )</td>
<td>If ( \lim_{n \to \infty} a_n \neq 0 ) or DNE</td>
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<tr>
<td>Integral Test (IT): ( \sum_{N} a_n = f(n) \geq 0 ), where ( f(n) ) is continuous on ([N, \infty])</td>
<td>If ( \int_{N}^{\infty} f(n) , dn ) converges</td>
<td>Never</td>
<td>If ( \int_{N}^{\infty} f(n) , dn ) diverges</td>
</tr>
<tr>
<td>&quot;P-Series&quot; Test (no acronym): ( \sum_{N} \frac{1}{n^p} )</td>
<td>If ( p &gt; 1 )</td>
<td>Never</td>
<td>If ( p \leq 1 )</td>
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<tr>
<td>Direct Comparison Test (DCT): ( \sum_{N} a_n ), where ( a_n \geq 0 )</td>
<td>If you can find ( b_n ) such that ( 0 \leq a_n \leq b_n ) and ( \sum_{N} b_n ) converges</td>
<td>Never</td>
<td>If you can find ( b_n ) such that ( 0 \leq b_n \leq a_n ) and ( \sum_{N} b_n ) diverges</td>
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<tr>
<td>Limit Comparison Test (LCT): ( \sum_{N} a_n ), where ( a_n &gt; 0 )</td>
<td>If you can find ( b_n \geq 0 ) such that ( \lim_{n \to \infty} \frac{a_n}{b_n} = c ) or ( 0 ) and ( \sum_{N} b_n ) converges</td>
<td>If ( \lim_{n \to \infty} \frac{a_n}{b_n} = 0 ) and ( \sum_{N} b_n ) diverges OR ( \sum_{N} b_n ) converges</td>
<td>If you can find ( b_n \geq 0 ) such that ( \lim_{n \to \infty} \frac{a_n}{b_n} = c ) or ( \infty ) and ( \sum_{N} b_n ) diverges</td>
</tr>
<tr>
<td>Ratio Test (RT): ( \sum_{N} a_n ), where ( a_n &gt; 0 )</td>
<td>If ( \lim_{n \to \infty} \frac{a_{n+1}}{a_n} &lt; 1 )</td>
<td>If ( \lim_{n \to \infty} \frac{a_{n+1}}{a_n} = 1 ) or DNE</td>
<td>If ( \lim_{n \to \infty} \frac{a_{n+1}}{a_n} &gt; 1 )</td>
</tr>
<tr>
<td>Nth Root Test (NRT): ( \sum_{N} a_n ), where ( a_n &gt; 0 )</td>
<td>If ( \lim_{n \to \infty} a_n^{1/n} &lt; 1 )</td>
<td>If ( \lim_{n \to \infty} a_n^{1/n} = 1 ) or DNE</td>
<td>If ( \lim_{n \to \infty} a_n^{1/n} &gt; 1 )</td>
</tr>
<tr>
<td>Absolute Convergence Test (ACT): Applies to any series with positive and negative terms.</td>
<td>If ( \sum_{N}</td>
<td>a_n</td>
<td>) converges, then ( \sum_{N} a_n ) converges</td>
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<tr>
<td>Alternating Series Test (AST): ( \sum_{N} (-1)^n u_n )</td>
<td>If all three things are true: 1. ( u_n &gt; 0 ) 2. ( u_n \geq u_{n+1} ) 3. ( \lim_{n \to \infty} u_n = 0 )</td>
<td>If either condition 1 or 2 is not met</td>
<td>If condition 3 is not met (see NTT)</td>
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</tbody>
</table>
- If a series passes the ACT, then it is said to converge "absolutely." If it does not pass the ACT, it may still converge, but only "conditionally." The only way we know to check conditional convergence is the AST.

- The Absolute Convergence Test will **NEVER** tell you if a series diverges.

- The $n^{th}$ Term Test will **NEVER** tell you if a series converges.

- The only two ways we know to find the actual sum of a convergent series is to use the Geometric Series Test, or to find the limit of $S_n$ (as in the Telescoping Series Test). Other tests may tell you if a series converges, but not what number the series converges to.

- There are Direct Comparison and Limit Comparison Tests for Integrals too. Don't confuse them.

- The "P-Series" Test *only* applies to series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$, *not* to anything else. For example:

  $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a P-Series, but $\sum_{n=1}^{\infty} \frac{1}{n+1}$ is not.