1. (Spring 2011 Exam 2) For Matrix

\[
A = \begin{bmatrix}
1 & 2 & -1 \\
-2 & 0 & 2 \\
-1 & 2 & 0
\end{bmatrix}
\]

(a) Find a basis for ker A.
(b) Find a basis for corng A.
(c) How do results in (a) and (b) be consistent with the Fundamental Theorem of Linear Algebra?
(d) Find conditions on \( b \) such that \( Ax = b \) has at least one solution

2. (Spring 2014 Exam 2) Let Matrix \( Q = (u_1, u_2, u_3, u_4) \) be a \( 4 \times 4 \) orthogonal matrix,

\[
B = \begin{bmatrix}
1 & 3 \\
2 & 1 \\
0 & 1
\end{bmatrix}
\]

and \( A = (u_1, u_2)B^T \).

(a) Find a basis for rng B.
(b) Prove that rng A=span \{u_1, u_2\}.
(c) Find a basis for cokerA. How does results in (b) and (c) illustrates the fundamental theorem of linear algebra?
(d) Find conditions on \( u_1, u_2 \) and \( b \), such that \( Ax = b \) has at least one solution.

3. Find the dimension of and the basis for the subspace spanned by the following column vectors:

\[
(1, 0, 1, 0)^T, (1, 0, 0, 1)^T, (2, 2, 1, 0)^T, (1, 2, 3, -3)^T.
\]

4. (Spring 2007 Exam 2) For this problem, assume \( A \) is a \( 3 \times 2 \) matrix, and consider the system \( Ax = b \).

(a) State the Fundamental Theorem of Linear Algebra.
(b) Under what conditions on \( A \) and \( b \) will the system have exactly one solution? Briefly (no more than 1-2 phrases or sentences) explain how these conditions are related to the Fundamental Theorem.
(c) now, suppose $\text{coker}(A) = \text{span} (2, 1, 0)^T, (1, 2, 1)^T$. What conditions must $b$ satisfy for at least one solution to exist?

(d) Assuming $\text{coker}(A)$ is given in part (c) what is $\text{rank}(A)$? What is $\dim(\text{corange}(A))$? What is $\text{rng}(A)$?

5. Let $A$ be a $m \times n$ matrix of rank $r$. Suppose $v_1, ..., v_n$ are basis for $R^n$ such that $v_{r+1}, ..., v_n$ forms a basis for $\ker A$. Prove that $w_1 = Av_1, ..., w_r = Av_r$ form a basis for $\text{rng} A$. 