1. Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$. Find a $100(1 - \alpha)\%$ confidence interval for $\sigma^2$ based on the sample variance and a $\chi^2$ critical value(s).

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from the exponential distribution with rate $\lambda$. Derive a $100(1 - \alpha)\%$ confidence interval for $\lambda$, based on $\bar{X}$, in terms of $\chi^2$-critical values.

3. Suppose that $X_1, X_2, \ldots, X_n \overset{iid}{\sim} \exp(rate = \lambda)$.
   
   (a) Give a test of size $\alpha$ for $H_0 : \lambda \leq \lambda_0$ versus $H_1 : \lambda > \lambda_0$ based on $X_{(1)}$, the minimum of the sample.
   
   (b) Find the power function for your test from part (a).

4. Suppose that $X_1, X_2, \ldots, X_n \overset{iid}{\sim} \exp(rate = \lambda)$.
   
   Find a test of size $\alpha$ for $H_0 : \lambda = \lambda_0$ versus $H_1 : \lambda > \lambda_0$ based on the sample mean $\bar{X}$. Give your test in terms of a $\chi^2$-critical value.

5. $X_1, X_2, \ldots, X_n \overset{iid}{\sim} \text{unif}(0, \theta)$. Find an MME (method of moments estimator) of $\theta$.

6. Let $X_1, X_2, \ldots, X_n$ be a random sample from the $N(\mu, \sigma^2)$ distribution. Find MMEs (method of moments estimators) for $\mu$ and $\sigma^2$.

7. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the Poisson distribution with rate $\lambda$.
   
   Find the MLE (maximum likelihood estimator) for $\lambda$.

8. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from the $\text{Pareto}(\gamma)$ distribution.
   
   (a) Find the MLE (maximum likelihood estimator) for $\gamma$.
   
   (b) Find an unbiased estimator of $\gamma$ based on the MLE from part (a).
   
   (c) [Required for 5520 only] Show that your MLE is a consistent estimator of $\gamma$.

9. [Required for 5520 Students Only] Consider a random sample of size $n_1 = 9$ from the $N(\mu_1, \sigma_1^2)$ distribution and an independent random sample of size $n_2 = 12$ from the $N(\mu_2, \sigma_2^2)$ distribution. Suppose that the variances are unknown but, for some crazy reason you do know that $\sigma_1^2 = 3\sigma_2^2$. Define a random variable that has a $t$-distribution that can be used to find a $100(1 - \alpha)\%$ confidence interval for $\mu_1 - \mu_2$. 