Newton’s Method
An iterative method for approximating the solutions to \( f(x) = 0 \).
\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
\]

Sigma Notation
\[
\sum_{i=1}^{n} c = nc
\]
\[
\sum_{i=m}^{n} a_i = c \sum_{i=m}^{n} a_i
\]
\[
\sum_{i=m}^{n} (a_i \pm b_i) = \sum_{i=m}^{n} a_i \pm \sum_{i=m}^{n} b_i
\]

Indefinite Integral
The set of all antiderivatives of a function \( f \) with respect to \( x \) is
\[
\int f(x) \, dx = F(x) + C.
\]

Definite Integral
If \( f \) is defined on \([a, b]\), the definite integral of \( f \) from \( a \) to \( b \) is
\[
\int_{a}^{b} f(x) \, dx = \lim_{\Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*) \Delta x_i
\]
where \( \Delta x_i \) is the length of the \( i \)th subinterval, and \( x_i^* \) is a sample point in the \( i \)th subinterval. If the limit exists, then \( f \) is integrable on \([a, b]\).

If \( f \) is integrable on \([a, b]\) then
\[
\int_{a}^{b} f(x) \, dx = \lim_{n \to \infty} R_n = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x
\]
where \( \Delta x = (b-a)/n \) and \( x_i = a + i \Delta x \).

Properties of the Definite Integral
1. \( \int_{a}^{a} f(x) \, dx = 0 \)
2. \( \int_{a}^{b} f(x) \, dx = -\int_{b}^{a} f(x) \, dx \)
3. \( \int_{a}^{b} (f(x) \pm g(x)) \, dx = \int_{a}^{b} f(x) \, dx \pm \int_{a}^{b} g(x) \, dx \)
4. \( \int_{a}^{b} cf(x) \, dx = c \int_{a}^{b} f(x) \, dx \), where \( c \) is a constant
5. \( \int_{c}^{a} f(x) \, dx + \int_{a}^{b} f(x) \, dx = \int_{c}^{b} f(x) \, dx \)

Comparison Properties of the Definite Integral
1. If \( f(x) \geq 0 \) for \( a \leq x \leq b \), then \( \int_{a}^{b} f(x) \, dx \geq 0 \).
2. If \( f(x) \geq g(x) \) for \( a \leq x \leq b \), then \( \int_{a}^{b} f(x) \, dx \geq \int_{a}^{b} g(x) \, dx \).
3. If \( m \leq f(x) \leq M \) for \( a \leq x \leq b \), then
\[
m(b-a) \leq \int_{a}^{b} f(x) \, dx \leq M(b-a).
\]

* You need not memorize formulas marked with an asterisk.
Suppose $f$ is continuous on $[-a, a]$.

If $f$ is even, then $\int_{-a}^{a} f(x) \, dx = 2 \int_{0}^{a} f(x) \, dx$.

If $f$ is odd, then $\int_{-a}^{a} f(x) \, dx = 0$.

**Fundamental Theorem of Calculus, Part I**
If $f$ is continuous on $[a, b]$, then the function $g$ defined by

$$ g(x) = \int_{a}^{x} f(t) \, dt, \quad a \leq x \leq b $$

is an antiderivative of $f$, that is, $g'(x) = f(x)$ for $a < x < b$.

**Fundamental Theorem of Calculus, Part II (Evaluation Theorem)**
If $f$ is continuous on $[a, b]$, then

$$ \int_{a}^{b} f(x) \, dx = F(b) - F(a) $$

where $F$ is any antiderivative of $f$, that is, $F'(x) = f(x)$.

**Net Change Theorem**

$$ \int_{a}^{b} F'(x) \, dx = F(b) - F(a). $$

**Average Value of a Function**

$$ f_{\text{ave}} = \frac{1}{(b-a)} \int_{a}^{b} f(x) \, dx $$

**Mean Value Theorem for Integrals**
If $f$ is continuous on $[a, b]$, then there is a number $c$ in $[a, b]$ such that

$$ f(c) = f_{\text{ave}} = \frac{1}{(b-a)} \int_{a}^{b} f(x) \, dx. $$

**Substitution Rule for Definite Integrals**
If $g'$ is continuous on $[a, b]$ and $f$ is continuous on the range of $u = g(x)$, then

$$ \int_{a}^{b} f(g(x))g'(x) \, dx = \int_{g(a)}^{g(b)} f(u) \, du. $$

A function $f$ is called a **one-to-one function** if

$$ f(x_1) \neq f(x_2) \text{ whenever } x_1 \neq x_2. $$

**Inverse Functions**
Let $f$ be a one-to-one function with domain $A$ and range $B$. Then its **inverse function** $f^{-1}$ has domain $B$ and range $A$ and is defined by

$$ f^{-1}(y) = x \iff f(x) = y $$

for any $y$ in $B$. The graph of $f^{-1}$ is obtained by reflecting the graph of $f$ about the line $y = x$. 
Derivative of an Inverse Function
If \( f \) is a one-to-one differentiable function with inverse function \( f^{-1} \) and \( f'(f^{-1}(a)) \neq 0 \), then

\[
(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}. 
\]

Natural Logarithm

\[
\ln x = \int_{1}^{x} \frac{1}{t} \, dt, \quad x > 0
\]

\[
\ln 1 = 0 \quad \ln e = 1
\]

\[
\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}
\]

\[
\int \frac{du}{u} = \ln |u| + C
\]

Laws of Logarithms

\[
\ln(xy) = \ln x + \ln y \quad \ln(x^r) = r \ln x
\]

\[
\ln \left(\frac{x}{y}\right) = \ln x - \ln y \quad \ln \left(\frac{1}{x}\right) = -\ln x
\]