Find det(A).

$$A = \begin{bmatrix} 1 & 4 & 0 & 1 \\ 3 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 \\ 2 & 1 & 2 & 1 \end{bmatrix}$$

Solution:

We can calculate the derivative by Laplacian expansion. We choose an arbitrary row or column in which to start, then we work our way across or down the matrix yielding n different calculations.

$$|A| = 1 \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 2 & 1 \end{vmatrix} - 4 \begin{vmatrix} 3 & 0 & 1 \\ 2 & 0 & 1 \\ 2 & 2 & 1 \end{vmatrix} + 0 \begin{vmatrix} 3 & 0 & 1 \\ 2 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 3 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 1 & 2 \end{vmatrix}$$

We find this by expansion.

$$A = \begin{bmatrix} \cancel{1} & \cancel{4} & \cancel{0} & \cancel{1} \\ \cancel{3} & 0 & 0 & 1 \\ \cancel{2} & 1 & 0 & 1 \\ \cancel{2} & 1 & 2 & 1 \end{bmatrix}$$

The first term above is found by multiplying a_{11} by the determinant of the matrix formed by excluding the entries in the row and column of a_{11}.

$$1 \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

The second term is found similarly, but the signs of each term alternate (so we use a − sign).

This matrix shows the pattern of signs used in the expansion.

$$\begin{bmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 0 & 1 \\ 3 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 \\ 2 & 1 & 2 & 1 \end{bmatrix} \qquad -4\begin{vmatrix} 3 & 0 & 1 \\ 2 & 0 & 1 \\ 2 & 2 & 1 \end{vmatrix}$$

etcetera

We find that $|A| = (1)(2) - (4)(-2) + 0 - (1)(6)$

$$\Rightarrow \boxed{|A| = -4}$$

Now, there's an easier way to find $|A|$ for this problem.
Since we can choose the row or column in which to expand, we should choose the row or column with the most zero entries.

So, choosing the third column,

$$|A| = 0\begin{vmatrix} 3 & 0 & 1 \\ 2 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 4 & 1 \\ 2 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix} + 0\begin{vmatrix} 1 & 4 & 1 \\ 3 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & 4 & 1 \\ 3 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= -2\left[-(4)\big((3)(1)-(1)(2)\big) + 0 - (1)\big((1)(1)-(1)(3)\big)\right]$$

$$= -2\left[-4+2\right]$$

$$= 4$$