# Exam Review Chapters $5, 6, 7$

1. (Fall 2011 Final #7) Consider the matrix:

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

   (a) Find all the eigenvalues of $A$.

   (b) For each eigenvalue that you found in part (a), find a basis for the corresponding eigenspace.

   (c) Find the general solution of $\vec{x}' = A\vec{x}$.

2. (Fall 2011 Final #8) Consider the following system of nonlinear equations:

$$\left\{ \begin{array}{rcl} \frac{dx}{dt} & = & y(x-1) \\ \frac{dy}{dt} & = & (x-y)(x+y-1) \end{array} \right\}$$

   (a) Find all the equilibrium points.

   (b) Classify the stability of each equilibrium point.

3. (Fall 2010 Final #2) Consider the system

$$\dot{x} = 20 - x^2 - y^2$$
$$\dot{y} = 8 - xy$$

   (a) Find the equilibrium points for this system.

   (b) Classify the stability of each equilibrium point.

   (c) Sketch the phase plane, showing all equilibria, nullclines, and some possible solution curves.

4. (Fall 2010 Final and assorted) Answer the following TRUE or FALSE.

   (a) Let $\lambda_1,\ \lambda_2,\ \lambda_3$ be the eigenvalues of

$$A = \begin{bmatrix} a & b & c \\ 0 & 0 & d \\ 0 & 0 & e \end{bmatrix}$$

   Then $\lambda_1 \lambda_2 \lambda_3 = 0$ and $\lambda_1 + \lambda_2 + \lambda_3 = (a + e)$.

(b) The second order ODE $y'' + \omega^2 y = G(y)$ is integrable (where $G(y)$ is a nonlinear function of $y$).

(c) Let $\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$ be a $2 \times 2$ nonlinear system. Let $(x_e, y_e)$ be an equilibrium solution to this system with a linearization that results in the eigenvalues $\lambda_1 = 2i$, $\lambda_2 = -2i$. It follows that the behavior of the nonlinear system in the local neighborhood of the equilibrium point is a center.

(d) Given the system $\vec{x}' = A\vec{x}$, answer each of the following either TRUE or FALSE

i. $\vec{x} = \vec{0}$ is the only equilibrium point.

ii. The point $\vec{0}$ is a sink if $A$ has a negative (real) eigenvalue.

iii. $A$ has $n$ eigenvalues and $n$ linearly independent eigenvectors.

iv. Assume $A$ is a $3 \times 3$ matrix. If $A$ has one eigenvalue $\lambda_1 = 2$ and a double eigenvalue $\lambda_{2,3} = 1$, then the general solution contains a generalized eigenvector if the geometric multiplicity of $\lambda_{2,3}$ is one.

(e) A limit cycle is attracting if the eigenvalue of the limit cycle is less than zero.

5. (Spring 2007, Exam 3) Consider the second order, linear, homogeneous ODE

$$x'' - x' - 6x = 0.$$

(a) Write the above equation in the matrix-vector form $\vec{w}' = A\vec{w}$.

(b) Find the general solution of the system derived in (a).

(c) Sketch the phase portrait of the system derived in (a).

(d) Find and classify the equilibria for the system derived in (a), and determine the behavior of solutions as $t \to \infty$.