Example 2, pg. 375

$$\underline{x}' = A\underline{x} = \begin{bmatrix} 6 & -1 \\ 5 & 4 \end{bmatrix} \underline{x}, \quad \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$|A - \lambda I| = (6-\lambda)(4-\lambda) - (-1)(5) = \lambda^2 - 10\lambda + 29 = 0$$

$$\lambda_{1,2} = 5 \pm 2i$$

$\underline{\lambda_1 = 5 + 2i}$

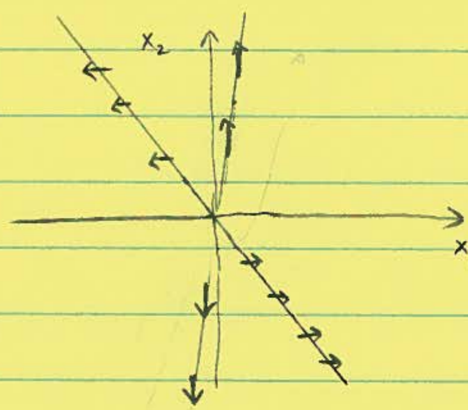
$$[A - \lambda_1 I] = \begin{bmatrix} 1-2i & -1 \\ 5 & -1-2i \end{bmatrix} \sim \begin{bmatrix} 5 & -1-2i \\ 5 & -1-2i \end{bmatrix} \sim \begin{bmatrix} 5 & -1-2i \\ 0 & 0 \end{bmatrix} \quad \text{so} \quad \underline{v_1} = \begin{bmatrix} 1 \\ 5 \end{bmatrix} + i \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

We can sketch trajectories in the x_1-x_2 phase plane.

The nullclines are,

$$x_1' = 0 = 6x_1 - x_2 \rightarrow x_2 = 6x_1$$

$$x_2' = 0 = 5x_1 + 4x_2 \rightarrow x_2 = -\frac{5}{4}x_1$$



When $x_1' = 0$, $x_2 = 6x_1 \rightarrow x_2' = 5x_1 + 4x_2$

$$= 5x_1 + 4(6x_1)$$

$$= 29x_1$$

So, when $x_1' = 0$, $x_2' = 29x_1$.

Similarly, when $x_2' = 0$, $x_2 = -\frac{5}{4}x_1 \rightarrow x_1' = 6x_1 -$

$$= 6x_1 +$$

$$= 7\tfrac{1}{4}x_1$$

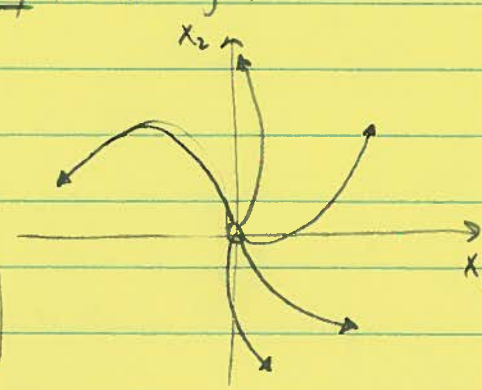
Since $\lambda_1 = \alpha + \beta i = 5 + 2i$,

we know solutions will spiral <u>away</u> from the origin.

From the nullcline arrows,

we see that trajectories will

spiral counterclockwise.



$\boxed{\underline{\text{CONTRAST}} \text{ with example 3, page 376.}}$