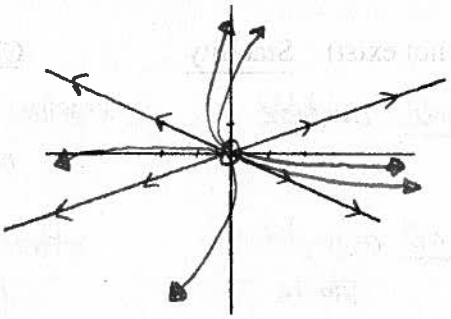
Stability classification of Linear Systems

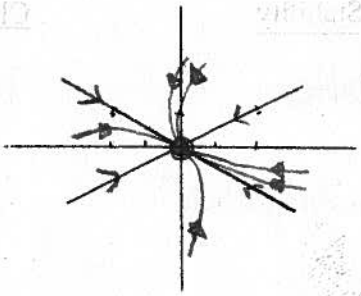
For cases 1-6, let $\vec{v}_1 = [-2, 1]^T$, $\vec{v}_2 = [2, 1]^T$.

Case 1: $\lambda_2 > \lambda_1 > 0$ | Stability | Classification



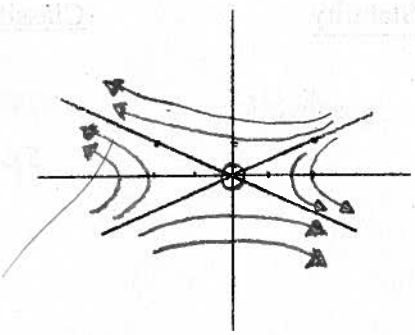
unstable | repelling node

Case 2: $\lambda_2 < \lambda_1 < 0$ | Stability | Classification



asymptotically stable | attracting node

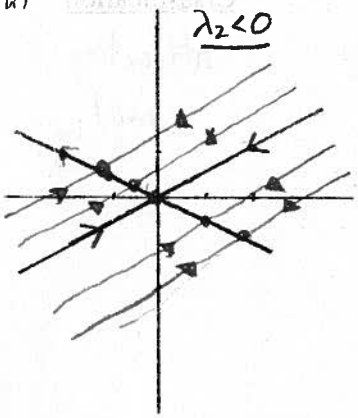
Case 3: $\lambda_1 > 0$, $\lambda_2 < 0$ | Stability | Classification



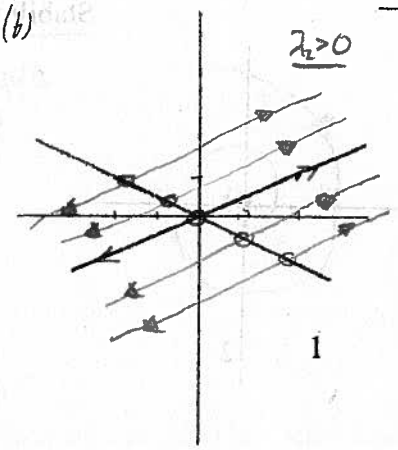
unstable | saddle

Case 4: $\lambda_1 = 0$, $\lambda_2 \neq 0$ | Stability | Classification

(a)



(b)



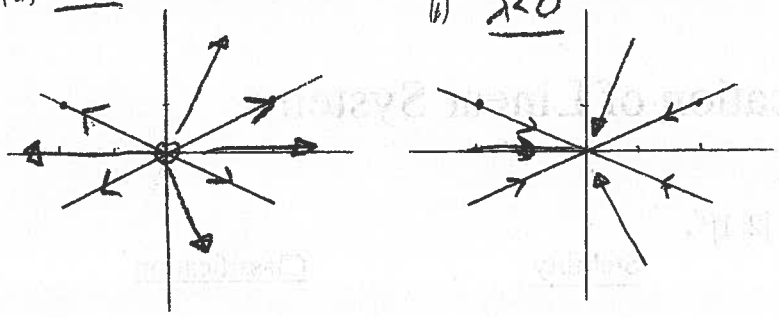
$\lambda_2 < 0$: asymptotically stable

$\lambda_2 > 0$: unstable

For cases 5 and 6, $\lambda \neq 0$.

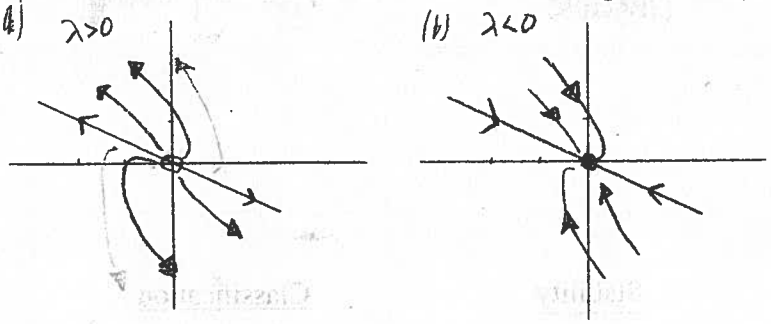
Case 5: $\lambda_1 = \lambda_2$ with two eigenvectors

	Stability	Classification
(a) $\lambda > 0$	$\lambda > 0$: unstable	repelling star node
(b) $\lambda < 0$	$\lambda < 0$: asymptotically stable	attracting star node



Case 6: $\lambda_1 = \lambda_2$ with one eigenvector (i.e., $\vec{v}_2$ does not exist)

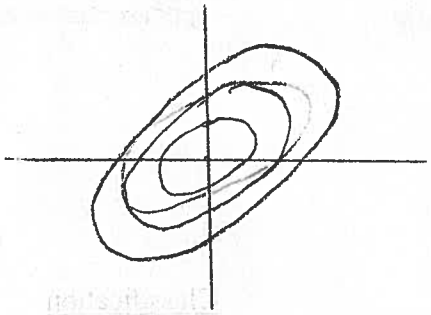
	Stability	Classification
(a) $\lambda > 0$	$\lambda > 0$: unstable	repelling degenerate node
(b) $\lambda < 0$	$\lambda < 0$: asymptotically stable	attracting degenerate node



Case A: λ complex, $\alpha = 0$

Stability: ~~unstable~~ stable or "neutrally stable"

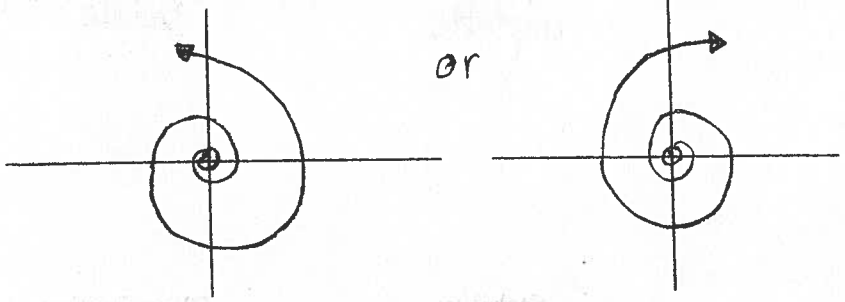
Classification: center



Case B: λ complex, $\alpha > 0$

Stability: unstable

Classification: repelling spiral



Case C: λ complex, $\alpha < 0$

Stability: asymptotically stable

Classification: attracting spiral