Homework set 1 — APMM5440, Fall 2006

From the textbook: 1.3, 1.4, 1.5.

**Problem 1:** Consider the set $\mathbb{R}^n$ equipped with the norm

$$||x||_p = \left( \sum_{j=1}^{n} |x_j|^p \right)^{1/p}.$$ 

(a) Prove that $|| \cdot ||_p$ is a norm for $p = 1$.

(b) Prove that $|| \cdot ||_p$ is a norm for $p = 2$.

(c) Prove that $\lim_{p \to \infty} ||x||_p = \max_{1 \leq j \leq n} |x_j|$.

**Problem 2:** Set $I = [0, 1]$ and consider the set $X$ consisting of all continuous functions on $I$. Define an addition and a scalar multiplication operator that make $X$ a normed linear space.

(a) Which of the following candidates define a norm on $X$:

- $||f||_a = \sup_{0 \leq x \leq 1} |f(x)|$
- $||f||_b = \sup_{0 \leq x \leq 1/2} |f(x)|$
- $||f||_c = \sup_{0 \leq x \leq 1} |f(x)|^2$
- $||f||_d = 2 \sup_{0 \leq x \leq 1} |f(x)|$
- $||f||_e = \sup_{0 \leq x \leq 1} (1 + \cos x)|f(x)|$
- $||f||_f = |f(0)| + \sup_{0 \leq x \leq 1} |f(x)|$
- $||f||_g = |f(0)|$

(b) Prove that

$$||f|| = \int_0^1 |f(x)| \, dx$$

is a norm on $X$.

(c) Prove that with respect to the norm given in (b), the space $X$ is not complete.