Homework set 4 — APPM5440 Fall 2012

From the textbook: 2.2, 2.3, 2.4, 2.5.

**Problem 1:** Show that on any set $X$, uniform convergence implies pointwise convergence.

**Problem 2:** Let $X$ be a finite set. Show that pointwise convergence on $X$ implies uniform convergence.

**Problem 3:** Let $X$ be an infinite set. Construct a sequence of functions $f_n : X \to \mathbb{R}$ that converges pointwise, but does not converge uniformly.

**Problem 4:** Let $X = [0, \infty)$. Construct a sequence of functions $f_n : X \to \mathbb{R}$ that converges uniformly (and hence pointwise), but that does not converge in $L^2(X)$.

**Problem 5:** Let $X = [0, 1]$. Construct a sequence of functions $f_n : X \to \mathbb{R}$ that converges in $L^2(X)$ but such that the sequence of numbers $(f_n(x))_{n=1}^\infty$ does not converge for any $x \in X$. 