10. ASSIGNMENT 9
Due Wednesday, April 9

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(1) The best known explicit Runge-Kutta method is defined by the following formulas:

\[
\begin{align*}
    k_1 &= hf(x_n, y_n) \\
    k_2 &= hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1) \\
    k_3 &= hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2) \\
    k_4 &= hf(x_n + h, y_n + k_3) \\
    y_{n+1} &= y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)
\end{align*}
\]

Determine the region of absolute stability of this method. Calculate all intersections of the region with the real and imaginary axes.

(2) One seeks the solution of the eigenvalue problem

\[
\frac{d}{dx} \left( \frac{1}{1+x} \right) \frac{dy}{dx} + \lambda y = 0,
\]

with boundary conditions \(y(0) = y(1) = 0\) by integrating, for a few values of \(\lambda\), an equivalent system of two first order differential equations with initial values \(y(0) = 0\) and \(y'(0) = 1\), using trapezoidal method combined with Richardson’s extrapolation developed in a previous assignment. Taking \(\lambda\) in the range \([6.7, 6.8]\), compute the value of \(\lambda\) for which \(y(1) = 0\).

(3) Problem #54, page 459 in Atkinson’s book (a simple problem on changing boundary conditions).