1. Consider the bottom half \((y \leq 0)\) of a disc of radius 1\(m\). The half disc has a constant mass density of 3\(kg/m^2\).

(a) What is the total mass of the disc?
(b) Where is the center of mass of the disc?
(c) What is the moment of inertia of the half disc around the \(x\) axis?
(d) What is the moment of inertia of the half disc around the \(y\) axis?

2. Consider the region bounded by the curves \(y = \frac{1}{x}, y = \frac{3}{x}, y = \sqrt{x}, y = 2\sqrt{x}\) in the first quadrant.

(a) Sketch the region in the xy-plane (You must label all curves).
(b) Setup an integral in xy-space that represents the area of the region (use the order \(dydx\)).
(c) Use the transformations \(u = xy\) and \(v = \frac{y^2}{x}\) to transform the region and sketch it in uv-space (You must label all curves).
(d) Setup an integral in uv-space that represents the area of the original region in xy space (use the order \(dvdu\)).
(e) Evaluate one of the integrals.

3. Consider the integral \(\int_0^{2\pi} \int_0^{\sqrt{3}} \int_{r}^{1} rdzdrd\theta\).

(a) Sketch the region in the \(r-z\) plane and label all bounding curves. (You can buy this sketch for 5pts)
(b) Express this integral in \(drdzd\theta\) order.
(c) Write the integral to compute the volume of this region in spherical coordinates using the order \(d\rho d\phi d\theta\).
(d) Write the integral to compute the volume of this region in spherical coordinates using the order \(d\phi d\rho d\theta\).
(e) Evaluate one of the integrals.

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**Projections and distances**

\[ \text{proj}_A B = \left( \frac{A \cdot B}{A \cdot A} \right) A \]

\[ d = \frac{|\vec{P}\vec{S} \times \vec{v}|}{|v|} \]

\[ d = \left| \vec{P}\vec{S} \cdot \frac{n}{|n|} \right| \]

**Arc length, frenet formulas, and tangential and normal acceleration components**

\[ ds = |v| \frac{dt}{dt} \]

\[ \xi = \frac{dr}{ds} = \frac{\vec{v}}{|\vec{v}|} \]

\[ \vec{N} = \frac{dT/ds}{|dT/ds|} = \frac{dT/dt}{|dT/dt|} \]

\[ \vec{B} = \vec{T} \times \vec{N} \]

\[ \frac{dT}{ds} = \kappa \vec{N} \]

\[ \frac{d\vec{B}}{ds} = -\tau \vec{N} \]

\[ \kappa = \left| \frac{dT}{ds} \right| \]

\[ \kappa = \left| \frac{dvA}{ds} \right| = \left| \frac{f''(x)}{1 + (f'(x))^2} \right|^{\frac{3}{2}} = \frac{|x\dddot{y} - \dddot{x}y|}{|x^2 + y^3|^{\frac{3}{2}}} \]

\[ \tau = -\frac{d\vec{B}}{ds} \cdot \vec{N} \]

\[ \vec{a} = a_N \vec{N} + a_T \vec{T} \]

\[ a_T = \frac{d|v|}{dt} \]

\[ a_N = \kappa |v|^2 = \sqrt{|a|^2 - a_T^2} \]
Directional derivative, discriminant, and Lagrange multipliers

\[ \frac{df}{ds} = (\nabla f) \cdot u \quad f_{xx}f_{yy} - (f_{xy})^2 \quad \nabla f = \lambda \nabla g, \quad g = 0 \]

Polar coordinates \( x = r \cos \theta \quad y = r \sin \theta \quad r^2 = x^2 + y^2 \quad dA = dx \, dy = r \, dr \, d\theta \)

Cylindrical and spherical coordinates

<table>
<thead>
<tr>
<th>Cylindrical to Rectangular</th>
<th>Spherical to Cylindrical</th>
<th>Spherical to Rectangular</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = r \cos \theta )</td>
<td>( r = \rho \sin \phi )</td>
<td>( x = \rho \sin \phi \cos \theta )</td>
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<tr>
<td>( y = r \sin \theta )</td>
<td>( z = \rho \cos \phi )</td>
<td>( y = \rho \sin \phi \sin \theta )</td>
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<td>( z = z )</td>
<td>( \theta = \theta )</td>
<td>( z = \rho \cos \phi )</td>
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\[ dV = dx \, dy \, dz = \rho \sin \phi \, d\rho \, d\phi \, d\theta \]

Substitutions in multiple integrals

\[ \int \int_R f(x, y) \, dx \, dy = \int \int_G f(x(u, v), y(u, v)) \, |J(u, v)| \, du \, dv \quad \text{where} \quad J(u, v) = \frac{\partial x}{\partial u} \frac{\partial y}{\partial v} - \frac{\partial y}{\partial u} \frac{\partial x}{\partial v} \]

Mass, moments, and center of mass

\[ \text{Mass} \quad M = \int \int_R \delta \, dA \]

\[ \text{Moments} \quad M_x = \int \int_R y \, \delta \, dA \quad M_y = \int \int_R x \, \delta \, dA \quad \text{Center of mass} \quad \bar{x} = M_y / M \quad \bar{y} = M_x / M \]