1. Verify that \( y = e^{t^2} \int_0^t e^{-s^2} \, ds + e^{t^2} \) is a solution of the differential equation \( y' - 2ty = 1 \).

**Sol:**

\[
y' = \frac{d}{dt} \left[ e^{t^2} \int_0^t e^{-s^2} \, ds + e^{t^2} \right]
\]

\[
= e^{t^2} \frac{d}{dt} \left[ \int_0^t e^{-s^2} \, ds \right] + \frac{d}{dt} \left[ e^{t^2} \right] \int_0^t e^{-s^2} \, ds + \frac{d}{dt} e^{t^2}
\]

\[
= e^{t^2} \left[ e^{-t^2} \right] + 2te^{t^2} \int_0^t e^{-s^2} \, ds + 2te^{t^2}
\]

\[
= 1 + 2te^{t^2} \int_0^t e^{-s^2} \, ds + 2te^{t^2}.
\]

\[
y' - 2ty = 1 + 2te^{t^2} \int_0^t e^{-s^2} \, ds + 2te^{t^2} - 2t(e^{t^2} \int_0^t e^{-s^2} \, ds + e^{t^2}) = 1.
\]

Therefore, \( y = e^{t^2} \int_0^t e^{-s^2} \, ds + e^{t^2} \) satisfies the differential equation.

2. Sketch the direction field for \( y' = y - t \). What can you say about the long term behavior of the solution?

**Sol:** Long term behavior depends on the initial condition. For points starting above the line \( y = t + 1 \) solutions tend towards \( +\infty \) as \( t \to \infty \). Below the line \( y = t + 1 \) solutions tend towards \( -\infty \) as \( t \to \infty \).
3. Match the differential equations with their corresponding direction fields.

(i) \( y' = t + y^2 \)
(ii) \( y' = e^{t^{1/2}} \)
(iii) \( y' = 1 - y \)

**Sol:** (i) \( \iff \) (c)
(ii) \( \iff \) (b), no \( y \) dependence
(iii) \( \iff \) (a), no \( t \) dependence

4. Which of the following equations are separable?

(a) \( y' = ty^2 \)
(b) \( y' = ty + t \)
(c) \( y' = -\frac{t^2}{y} \)
(d) \( y = \sin(ty) \)
(e) \( y = \log(y') \)

**Sol:** (a), (b), (c) and (e) are separable.