**Problem 1:** Consider the following system of differential equations,

\[
\frac{dx}{dt} = x - x^2 - yx \\
\frac{dy}{dt} = 2y - y^2 - 2yx
\]

(a) Find and plot the nullclines and equilibrium points.

(b) Show the directions of the vector field between the nullclines, and sketch solution curves.

(c) Label equilibrium points as stable or unstable.

**Problem 2:** Consider the following matrices \( A \) and \( B \), and the vectors \( c \) and \( d \),

\[
A = \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 1 \\ 8 & 2 \\ 1 & 7 \end{pmatrix}, \quad c = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad d = \begin{pmatrix} -1 \\ 1 \end{pmatrix}
\]

(a) Compute the product \( AB \) and \( BA \). Which is not defined?

(b) Compute the absolute value of vector \( c \), or \( ||c|| \).

(c) Are vectors \( c \) and \( d \) orthogonal?
Problem 3: Solve the following algebraic system by using row operations,

\[
\begin{align*}
  x + y + 2z &= 2 \\
  3x - 4y &= 3 \\
  x - z &= 4
\end{align*}
\]

Problem 4: Use matrix and vector properties to simplify the following expressions, where \( A \), \( B \) are matrices and \( \alpha, \beta \) are vectors.

(a) Simplify \(((\alpha B)^T(\beta A)^T)^T - \alpha \beta AB\).

(b) Suppose that \( A = A^T \) and \( B = B^T \). Simplify \((A + B)(A + B)^T - (BA^T + AB^T)\).