A function $f$ is **even** if $f(-x) = f(x)$ for all $x$. 
A function $f$ is **odd** if $f(-x) = -f(x)$ for all $x$.

The **average rate of change** of $y = f(x)$ on $[x_1, x_2]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$ 

This is also the **slope of the secant line**.

**Informal Definition of Limit**

$$\lim_{x \to a} f(x) = L$$

if the values of $f(x)$ approach $L$ as $x$ approaches $a$.

**Squeeze Theorem**

If $f(x) \leq g(x) \leq h(x)$ when $x$ is near $a$ and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$, then

$$\lim_{x \to a} g(x) = L.$$ 

**Definition of Derivative**

The **derivative** of the function $f$ with respect to the variable $x$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}.$$ 

The **slope of the tangent line** at $x = a$ is $f'(a)$.

Alternate form:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$ 

A function is **differentiable** at $x = a$ if and only if the $f$ is continuous at $x = a$, and the left-hand and right-hand derivatives are equal.
Product Rule

\[(fg)' = f \cdot g' + g \cdot f'.\]

Quotient Rule

\[\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2}.\]

Chain Rule

\[(f \circ g)'(x) = f'(g(x)) \cdot g'(x).\]

Linearization of a Function

\[L(x) = f(a) + f'(a)(x - a)\]

Differential Change

\[dy = f'(x) \, dx.\]

Finding Absolute Extrema

Suppose \(f\) is continuous on a closed interval.

1. Find the values of \(f\) at the critical numbers.
2. Find the values of \(f\) at the endpoints.
3. The largest value is the absolute maximum. The smallest value is the absolute minimum.

A critical number is a value \(c\) in the domain of \(f\) such that \(f'(c) = 0\) or \(f'(c)\) does not exist.

An inflection point on the graph of \(f\) is a point where \(f\) is continuous and the concavity of \(f\) changes.

Guidelines for Graphing \(y = f(x)\)

1. Identify the domain and symmetry of \(f\).
2. Find the \(x\) and \(y\) intercepts of \(f\).
3. Find horizontal asymptotes: \(\lim_{x \to \pm\infty} f(x) = L\).
   
   Find vertical asymptotes: \(\lim_{x \to a^-} f(x) = \pm\infty, \lim_{x \to a^+} f(x) = \pm\infty\).
4. Find \(y'\) and \(y''\). Identify the critical numbers where \(y' = 0\) or \(y'\) is undefined.
5. Determine the rise and fall of \(f(x)\). Identify local extrema.
6. Determine the concavity of \(f(x)\). Identify inflection points.
7. Summarize. Sketch the general shape of \(f\).
8. Plot critical numbers and intercepts. Sketch the curve.
Mean Value Theorem
If \( f \) is continuous on \([a, b]\) and differentiable on \((a, b)\), then there is a number \( c \) in \((a, b)\) such that
\[
f'(c) = \frac{f(b) - f(a)}{b-a}.
\]

Newton’s Method
An iterative method for approximating the solutions to \( f(x) = 0 \).

\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
\]

Sigma Notation
\[
\sum_{i=1}^{n} c = nc
\]
\[
\sum_{i=m}^{n} c a_i = c \sum_{i=m}^{n} a_i
\]
\[
\sum_{i=m}^{n} (a_i \pm b_i) = \sum_{i=m}^{n} a_i \pm \sum_{i=m}^{n} b_i
\]
\[
\sum_{i=1}^{n} i = \frac{n(n+1)}{2}
\]
\[
\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}
\]
\[
\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2
\]

Area as the Sum of Approximating Rectangles
The area \( A \) of the region that lies under the graph of a continuous function \( f \) on \([a, b]\) is
\[
A = \lim_{n \to \infty} R_n = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x
\]
where \( \Delta x = (b-a)/n \) is the length of each subinterval, and \( x_i = a + i \Delta x \) is the right endpoint of the \( i \)th subinterval.

Definite Integral as a Limit of Riemann Sums
If \( f \) is a function defined on \([a, b]\), the definite integral of \( f \) from \( a \) to \( b \) is
\[
\int_{a}^{b} f(x) \, dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*) \Delta x_i
\]
where \( \Delta x_i \) is the length of the \( i \)th subinterval \([x_{i-1}, x_i]\), and sample point \( x_i^* \) is any number in the \( i \)th subinterval.

Fundamental Theorem of Calculus, Part I
If \( f \) is continuous on \([a, b]\), then the function \( g \) defined by
\[
g(x) = \int_{a}^{x} f(t) \, dt, \quad a \leq x \leq b
\]
is an antiderivative of \( f \), that is, \( g'(x) = f(x) \) for \( a < x < b \).

Fundamental Theorem of Calculus, Part II
If \( f \) is continuous on \([a, b]\), then
\[
\int_{a}^{b} f(x) \, dx = F(b) - F(a)
\]
where \( F \) is any antiderivative of \( f \), that is, \( F'(x) = f(x) \).

Net Change Theorem
\[
\int_{a}^{b} f'(x) \, dx = F(b) - F(a).
\]

Average Value of a Function
\[
f_{ave} = \frac{1}{(b-a)} \int_{a}^{b} f(x) \, dx
\]
Mean Value Theorem for Integrals
If \( f \) is continuous on \([a, b]\), then there is a number \( c \) in \([a, b]\) such that
\[
f(c) = f_{\text{ave}} = \frac{1}{b - a} \int_a^b f(x) \, dx.
\]

A function \( f \) is a one-to-one function if
\[
f(x_1) \neq f(x_2) \quad \text{whenever} \quad x_1 \neq x_2.
\]

Inverse Functions
Let \( f \) be a one-to-one function with domain \( A \) and range \( B \). Then its inverse function \( f^{-1} \) has domain \( B \) and range \( A \) and is defined by
\[
f^{-1}(y) = x \iff f(x) = y
\]
for any \( y \) in \( B \). The graph of \( f^{-1} \) is obtained by reflecting the graph of \( f \) about the line \( y = x \).

Natural Logarithm and Natural Exponential Function
\[
\ln x = \int_1^x \frac{1}{t} \, dt, \quad x > 0
\]
\[
e^y = x \iff x = \ln y
\]
\[
e^{\ln x} = x \quad \ln e^x = x
\]
\[
e = \lim_{n \to 0} (1 + x)^{1/n} = \lim_{n \to \infty} \left( 1 + \frac{1}{n} \right)^n
\]

Laws of Logarithms
\[
\ln(xy) = \ln x + \ln y \quad \ln(x^r) = r \ln x
\]
\[
\ln \left( \frac{x}{y} \right) = \ln x - \ln y \quad \ln \left( \frac{1}{x} \right) = -\ln x
\]

General Logarithmic Functions
\[
\log_a x = y \iff a^y = x
\]
\[
\log_a x = \frac{\ln x}{\ln a}
\]
\[
a^{\log_a x} = x \quad \log_a a^x = x
\]

Derivative Formulas
\[
\frac{d}{dx} (x^n) = nx^{n-1}
\]
\[
\frac{d}{dx} (e^x) = e^x
\]
\[
\frac{d}{dx} (\ln x) = \frac{1}{x}
\]
\[
\frac{d}{dx} (e^x) = e^x
\]
\[
\frac{d}{dx} (\sin x) = \cos x
\]
\[
\frac{d}{dx} (\cos x) = -\sin x
\]
\[
\frac{d}{dx} (\tan x) = \sec^2 x
\]
\[
\frac{d}{dx} (\cot x) = -\csc^2 x
\]
\[
\frac{d}{dx} (\csc x) = -\csc x \cot x
\]
\[
\frac{d}{dx} (\sec x) = \sec x \tan x
\]
\[
\frac{d}{dx} (\ln a) = 1
\]
Integral Formulas

\[\int u^n \, du = \frac{u^{n+1}}{n+1} + C \quad \int e^u \, du = e^u + C\]
\[\int \frac{du}{u} = \ln |u| + C \quad \int a^u \, du = \frac{a^u}{\ln a} + C\]

Law of Exponential Change

\[y(t) = y_0 e^{kt}\]

Newton’s Law of Cooling

\[T(t) - T_S = (T_0 - T_S)e^{kt}\]

Inverse Trigonometric Functions

\[\sin^{-1} x = \theta \iff \sin \theta = x \quad \text{and} \quad \theta \in [-\pi/2, \pi/2]\]
\[\cos^{-1} x = \theta \iff \cos \theta = x \quad \text{and} \quad \theta \in [0, \pi]\]
\[\tan^{-1} x = \theta \iff \tan \theta = x \quad \text{and} \quad \theta \in (-\pi/2, \pi/2)\]
\[\cot^{-1} x = \theta \iff \cot \theta = x \quad \text{and} \quad \theta \in (0, \pi)\]
\[\sec^{-1} x = \theta \iff \sec \theta = x \quad \text{and} \quad \theta \in [0, \pi/2) \cup [\pi, 3\pi/2)\]
\[\csc^{-1} x = \theta \iff \csc \theta = x \quad \text{and} \quad \theta \in (0, \pi/2] \cup (\pi, 3\pi/2]\]

Derivatives of Inverse Trigonometric Functions

\[\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx} (\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}\]
\[\frac{d}{dx} (\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \quad \frac{d}{dx} (\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}\]
\[\frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2} \quad \frac{d}{dx} (\cot^{-1} x) = -\frac{1}{1+x^2}\]

Trigonometric Identities

\[\sin^2 \theta + \cos^2 \theta = 1 \quad \sin 2\theta = 2 \sin \theta \cos \theta\]
\[\tan^2 \theta + 1 = \sec^2 \theta \quad \cos 2\theta = 2 \cos^2 \theta - 1\]
\[1 + \cot^2 \theta = \csc^2 \theta \quad \cos 2\theta = 1 - 2 \sin^2 \theta\]

Hyperbolic Functions

\[\sinh x = \frac{e^x - e^{-x}}{2} \quad \cosh x = \frac{e^x + e^{-x}}{2}\]
\[\cosh x = \frac{1}{\sinh x} \quad \sech x = \frac{1}{\cosh x}\]
\[\tanh x = \frac{\sinh x}{\cosh x} \quad \coth x = \frac{\cosh x}{\sinh x}\]
\[\sinh(-x) = -\sinh x \quad \cosh^2 x - \sinh^2 x = 1\]
\[\cosh(-x) = \cosh x\]

* You need not memorize formulas marked with an asterisk.
### Derivatives of Hyperbolic Functions

\[
\frac{d}{dx} (\sinh x) = \cosh x \quad \frac{d}{dx} (\cosh x) = \sinh x \quad \frac{d}{dx} (\tanh x) = \text{sech}^2 x
\]

\[
\frac{d}{dx} (\csc h x) = -\csc h x \coth x \quad \frac{d}{dx} (\text{sech} x) = -\text{sech} x \tanh x
\]

\[
\frac{d}{dx} (\text{coth} x) = -\csc h^2 x
\]

### Strategies for Finding Limits

1. Simplify and try substitution.
2. Indeterminate Forms (0/0) and (∞/∞).
   - (a) L'Hôpital’s Rule.
   - (b) Factor and cancel.
   - (c) Multiply by a conjugate and cancel.
   - (d) Use \( \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \).
3. Infinite limit \((k/0)\) where \(k\) is a non-zero constant.
   - (a) Evaluate left-hand and right-hand limits separately.
   - (b) Determine whether answer is \(+\infty\) or \(-\infty\) or undefined.
4. Piecewise and absolute value functions.
   - (a) Evaluate left-hand and right-hand limits separately.
5. Try Squeeze Theorem.

### Inverse Hyperbolic Functions

\[
y = \sinh^{-1} x \iff \sinh y = x \quad y = \csc h^{-1} x \iff \csc h y = x
\]

\[
y = \cosh^{-1} x \iff \cosh y = x \quad y = \text{sech}^{-1} x \iff \text{sech} y = x
\]

\[
y = \tanh^{-1} x \iff \tanh y = x \quad y = \text{coth}^{-1} x \iff \text{coth} y = x
\]

### Derivatives of Inverse Hyperbolic Functions

\[
\frac{d}{dx} (\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}} \quad \frac{d}{dx} (\csc h^{-1} x) = \frac{-1}{|x|\sqrt{x^2+1}}
\]

\[
\frac{d}{dx} (\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}} \quad \frac{d}{dx} (\text{sech}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}
\]

\[
\frac{d}{dx} (\tanh^{-1} x) = \frac{1}{1-x^2} \quad \frac{d}{dx} (\text{coth}^{-1} x) = \frac{1}{1-x^2}
\]

### L'Hôpital’s Rule

\[
\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}
\]

for indeterminate forms \(0/0\) or \(\infty/\infty\).

### Geometry Formulas

- Distance formula: \(d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\)
- Circumference of a circle: \(C = 2\pi r\)
- Area of a circle: \(A = \pi r^2\)
- Area of a trapezoid: \(A = \frac{1}{2}h(b_1 + b_2)\)
- Volume of a sphere: \(V = \frac{4}{3}\pi r^3\)
- Surface area of a sphere: \(A = 4\pi r^2\)
- Volume of a cylinder: \(V = \pi r^2 h\)
- Surface area of a cylinder: \(A = 2\pi rh + 2\pi r^2\)
- Volume of a cone: \(V = \frac{1}{3}\pi r^2 h\)

* You need not memorize formulas marked with an asterisk.