1. A body with mass $m = \frac{1}{2}$ kg is attached to the end of a spring that is stretched 2 m by a force of 100 N. It is set in motion with initial position $x_0 = 1$ m and initial velocity $v_0 = -5$ m/s. Find the position function of the body as well as the amplitude and period of oscillation.

2. Express the position function from problem 1 in the form

$$x(t) = A \cos (\omega_0 t - \delta).$$

3. Solve the initial value problem

$$y'' - 3y' + 2y = 3e^{-t} - 10 \cos 3t,$$

$$y(0) = 1, \quad y'(0) = 2.$$

4. Find the general form of a particular solution of

(a) $y''' + 9y' = t \sin t + t^2 e^{2t}$

(b) $y'' - 2y' + 2y = e^t \sin t$

(c) $y'' + 4y = 3t \cos 2t$

5. Find a particular solution of the equation

$$y'' + y = \tan t.$$

**Hint:** $\int \sec t \, dt = \ln |\sec t + \tan t| + C$.

6. Use variation of parameters to find a particular solution of

$$y'' - 4y' + 4y = 2e^{2t}$$

7. Find the eigenvalues and corresponding eigenspaces for the matrix

$$\begin{bmatrix} 3 & -8 \\ 2 & 3 \end{bmatrix}$$

8. Find the eigenvalues and corresponding eigenspaces for the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$
9. Show that $A$ and $A^T$ have the same eigenvalues. Do they necessarily have the same eigenvectors? Explain.

10. Let $A$ be an $n \times n$ matrix. Prove that $A$ is singular if and only if $\lambda = 0$ is an eigenvalue of $A$.

11. An $n \times n$ matrix $A$ is said to be idempotent if $A^2 = A$. Show that if $\lambda$ is an eigenvalue of an idempotent matrix, then $\lambda$ must be either 0 or 1.

12. Let $\lambda$ be a nonzero eigenvalue of $A$ and let $\vec{x}$ be an eigenvector belonging to $\lambda$. Show that $A^m \vec{x}$ is also an eigenvector belonging to $\lambda$ for $m = 1, 2, 3, \ldots$. 