1 Problems from Strogatz

- (10 points each) 5.1.1, 5.2.8.
- (10 points each) 6.1.13, 6.3.1, 6.3.7 (only for 6.3.1), 6.3.13.

2 Other problems

1. (Box-counting dimension - theoretical)

   (a) (10 points) Suppose we remove the middle open third from [0, 1]. Then we remove the middle fifth from the two remaining intervals. Then we remove the middle third of every remaining interval, and so on, alternating between removing the middle third and the middle fifth at every step. Find the box counting dimension of the resulting Cantor set.

   (b) (5 points) More generally, suppose we construct a Cantor set by alternating between removing the middle \(x\)'th and the middle \(y\)'th [where \(x, y \in (0, 1)\) represent fractions]. Let \(D_{xy}\) be the box counting dimension of this Cantor set. Also let \(D_z\) be the box counting dimension of the Cantor set obtained by always removing the middle \(z\)'th. Show that

   \[
   \frac{1}{D_{xy}} = \frac{1}{2} \left( \frac{1}{D_x} + \frac{1}{D_y} \right). \tag{1}
   \]

2. (Box-counting dimension - numerical)

   Consider the two-dimensional map

   \[
   x_{n+1} = \sin(by_n) + c\sin(bx_n), \tag{2}
   \]
   \[
   y_{n+1} = \sin(ax_n) + d\sin(ay_n), \tag{3}
   \]

   with

   \[a = 2.4, \quad b = 1.4, \quad c = 0.5, \quad d = 0.5. \tag{4}\]

   (a) (10 points) Pick an initial condition \(x_0 \in (0, 1), y_0 \in (0, 1)\). Iterate the map a large number of times (100000 or more), and plot the points \(\{(x_n, y_n)\}\) to produce a picture of the attractor for this map. You might want to discard the first iterations to allow the orbit to settle into the attractor. Also, make the points small so the attractor structure can be seen.

   (b) (5 points) Estimate numerically the box-counting dimension of the attractor by using the orbit \(\{(x_n, y_n)\}\) you collected in part (a). [Calculate \(N(\varepsilon)\) for various values of \(\varepsilon\) and find the slope of \(\log(N(\varepsilon))\) versus \(\log(1/\varepsilon)\)\].