INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. This exam is worth 100 points. Box your final answers. Write neatly, top to bottom, left to right, one problem per page. A correct answer with incorrect or no supporting work may receive no credit. **SHOW ALL WORK**

1. (13 points; 3,3,3,4) Solve for $x$, simplify (Consider $x$ as a real number and $0 \leq x \leq 2\pi$ for trig functions):

   (a) $-9^2 = \sqrt{-8} + x$  
   (b) $x \cdot 8^{1/2} = \sqrt{2}$  
   (c) $\left[ \frac{4-3\cdot2+3+\frac{1}{2}}{(\frac{1}{2})^2} \right]^2 = x/2$  
   (d) $\sin 2x = \cos x$.

Solution:

(a) $x = -81 + 2 = -79$  
(b) $\frac{\sqrt{2}}{\sqrt{8}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$  
(c) $x = 2 \left( \frac{16}{5} \right) = 2 \cdot 2 \cdot 16 = 2 \cdot 32 = 64$

(d) 

$$\sin 2x = \cos x$$
$$2 \sin x \cos x = \cos x$$
$$\cos x(2 \sin x - 1) = 0$$
$$\cos x = 0 \quad \text{or} \quad \sin x = \frac{1}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$
2. (22 points; 3,3,5,4,7) Consider the following functions:

\[ f(x) = \frac{|x+1|}{x} \quad \text{and} \quad g(x) = \sqrt{x^2 - 1} \quad \text{and} \quad h(x) = \begin{cases} 
  x^2 & x \leq -1 \\
  -x^2 + 1 & -1 < x < 1 \\
  -1 + x^2 & x > 1 
\end{cases} \]

(a) What is the domain of \( g(x) \).  
(b) Is \( g(x) \) even, odd, neither, or both?  
(c) What is \( h \circ g(x) \)?  
(d) State the domain of \( h \circ g(x) \).  
(e) Sketch \( f(x) \).

**Solution:**

(a) \( x^2 - 1 \geq 0 \\
\Rightarrow x^2 \geq 1 \\
\Rightarrow |x| \geq 1 \\
\Rightarrow (-\infty, -1] \cup [1, \infty) \\

(b) \( f(x) = \sqrt{x^2 - 1} \\
f(-x) = \sqrt{(-x)^2 - 1} \\
f(-x) = \sqrt{x^2 - 1} = f(x) \\
f(x) \text{ is Even} \\

(c) Since \( h \circ g(x) \) excludes values \((-1, 1)\) from its domain we have:

\[ h \circ g(x) = \begin{cases} 
  (\sqrt{x^2 - 1})^2 & , x \leq -1 \\
  \emptyset & , -1 < x < 1 \\
  -1 + (\sqrt{x^2 - 1})^2 & , x > 1 
\end{cases} \]

\[ h \circ g(x) = \begin{cases} 
  x^2 - 1 & , x \leq -1 \\
  \emptyset & , -1 < x < 1 \\
  x^2 - 2 & , x > 1 
\end{cases} \]

(d) The \( h(x) \) function excludes 1 from its domain. The domain of \( h \circ g(x) \) is the intersection of the domains of \( h(x) \) and \( g(x) \) or \((-\infty, -1] \cup (1, \infty)\)

(e) \( f(x) = \frac{|x+1|}{x} \) can be described piecewise:

\[ f(x) = \begin{cases} 
  \frac{x+1}{x} & , x \geq -1 \\
  \frac{x-1}{x} & , x < -1 
\end{cases} \]

or

\[ f(x) = \begin{cases} 
  1 + \frac{1}{x} & , x \geq -1 \\
  -1 - \frac{1}{x} & , x < -1 
\end{cases} \]

These are the graphs of \( \frac{1}{x} \) shifted and rotated.

![Graph of f(x)](image)
3. (18 points; 3,3,4,8) Consider the following graph of a functional relationship $f(x)$ with secant line $PQ$:

(a) What are the coordinates of point $P$?  
(b) What are the coordinates of point $Q$?

(c) What is the slope of line $PQ$?  
(d) Suppose $f(x) = \frac{1}{x}$, then find and simplify $\frac{f(a + h) - f(a)}{h}$.

**Solution:**

(a) $[a, f(a)]$

(b) $[a + h, f(a + h)]$

(c) $m = \frac{f(a + h) - f(a)}{h}$

(d) $\frac{1}{a + h} - \frac{1}{a} = \frac{a - (a + h)}{ha(a + h)} = \frac{-h}{ha(a + h)} = \frac{-1}{a(a + h)}$
4. (8 points) Solve for $x$: \[
\frac{\sin x \cdot \sqrt{x^2 + 9} - 3 \sin x}{x^3} = 0.
\]

**Solution:** The only time a fraction can equal zero is when the numerator is zero and the denominator is non-zero.

\[
\sin x \cdot \sqrt{x^2 + 9} - 3 \sin x = 0
\]

\[
\sin x (\sqrt{x^2 + 9} - 3) = 0
\]

\[
\sin x = 0 \quad \text{or} \quad \sqrt{x^2 + 9} - 3 = 0
\]

\[
x = n\pi \quad \text{or} \quad \sqrt{x^2 + 9} = 3
\]

\[
x^2 + 9 = 9 \quad \implies \quad x^2 = 0 \quad \implies \quad x = 0
\]

$n\pi$ for integer n’s $\neq 0$ since x=0 is not a member of the functions domain.
5. (10 points) A truck can be rented from Basic Rental for $50 per day plus $0.20 per mile. Continental charges $20 per day plus $0.50 per mile to rent the same truck. What is the minimum number of miles that must be driven in a day to make the rental cost for Basic Rental a better deal than Continental’s?

**Solution:** Continental appears to have a better deal prior to the point where the deals are equivalent, which is when \( 50 + 0.20x = 20 + 0.50x \). For \( x \) amount of miles after this point, Basic Rental is better.

\[
\begin{align*}
50 + 0.20x &= 20 + 0.50x \\
30 &= 0.30x \\
x &= 30/0.30 \\
x &= 100
\end{align*}
\]

Therefore, Basic Rental is better [AFTER 100 miles], or at \( x > 100 \) miles.
6. (10 points) If the following solid has a volume of 21 cubic units, solve for $x$. All corners are 90° angles.

Solution:

\[
V = 3x(2x - 1) + x^2(x + 1) = 21 \\
6x^2 - 3x + x^3 + x^2 = 21 \\
x^3 + 7x^2 - 3x - 21 = 0 \\
x^2(x + 7) - 3(x + 7) = 0 \\
(x + 7)(x^2 - 3) = 0 \\
x = -7, -\sqrt{3}, \sqrt{3}
\]

The only valid answer for a length is $x = \sqrt{3}$
7. (19 points; 4,4,6,5) Answer the following:
(a) Describe the shape, size, and location of the graph of \(3x^2 + 3y^2 + 6x - 12y = 0\).

(b) If a 45-45-90 triangle has a leg of length \(\sqrt{2} + \sqrt{3}\), then how long is the hypotenuse?

(c) What single trigonometry function is equivalent to \(\tan \theta \sin \theta + \cos \theta\)? Explain.

(d) Solve for \(P\) given that \(T = \frac{A - P}{Pr}\).

Solution: (a)

\[
\begin{align*}
3x^2 + 3y^2 + 6x - 12y &= 0 \\
3x^2 + 6x + 3y^2 - 12y &= 0 \\
3(x^2 + 2x) + 3(y^2 - 4y) &= 0 \\
3(x^2 + 2x + 1) + 3(y^2 - 4y + 4) &= 3 + 12 \\
3(x + 1)^2 + 3(y - 2)^2 &= 15 \\
(x + 1)^2 + (y - 2)^2 &= 5
\end{align*}
\]

This is a circle located at \((-1, 2)\) with a radius of length \(\sqrt{5}\)

(b) The hypotenuse will be \(\sqrt{2}\) times the leg, thus: \(h = \sqrt{2}(\sqrt{2} + \sqrt{3}) = 2 + \sqrt{6}\)

alternatively, \(h^2 = 2(\sqrt{2} + \sqrt{3})^2 \implies h = \sqrt{10 + 4\sqrt{6}}\)

(c)

\[
\tan \theta \sin \theta + \cos \theta = \frac{\sin \theta}{\cos \theta} \sin \theta + \cos \theta \\
= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} \\
= \frac{1}{\cos \theta} \\
= \sec \theta
\]

(d)

\[
\begin{align*}
T &= \frac{A - P}{Pr} \\
TPr &= A - P \\
TPr + P &= A \\
P(Tr + 1) &= A \\
P &= \frac{A}{Tr + 1}
\end{align*}
\]
END of Exam