# TFOCS: Flexible First-order Methods for Rank Minimization

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# Typical problems: matrix completion

## Matrix completion

$$\min_X \|X\|_{\text{tr}} \quad \text{such that} \quad \mathcal{A}(X) = b, X \in \mathbb{R}^{n_1 \times n_2}.$$

$\|X\|_{\text{tr}}$ is the nuclear norm (sum of singular values).
$\mathcal{A} : \mathbb{R}^{n_1 \times n_2} \to \mathbb{R}^m$ is linear

$$\mathcal{A}(X) = \begin{bmatrix} \times & ? & ? & \times & ? \\ ? & \times & ? & \times & \times \\ \times & \times & ? & ? & ? \\ ? & \times & \times & \times & ? \\ ? & ? & \times & ? & \times \end{bmatrix}$$

If $m \ll n_1 \times n_2$, want prior on $X$. Convenient prior: $X$ is low-rank.

Variants:

$$\min_X \|X\|_{\text{tr}} \quad \text{such that} \quad \|\mathcal{A}(X) - b\|_2 \le \varepsilon$$

$$\min_X \|X\|_{\text{tr}} + \tau \|\mathcal{A}(X) - b\|_2^2$$

# Typical problems: RPCA

Robust PCA (one type):

## RPCA

$$\min_{L,S} \|L\|_{\text{tr}} + \lambda \|S\|_1 \quad \text{such that} \quad L + S = X, \mathcal{A}(X) = b$$

Idea: $X$ is composed of Low-rank and Sparse

May use $\mathcal{A} = I$

variants, e.g. AWGN noise:

## Stable Principal Component Pursuit

$$\min_{L,S} \|L\|_{\text{tr}} + \lambda \|S\|_1 \quad \text{such that} \quad \|\mathcal{A}(X) - b\|_2 \leq \varepsilon$$

or constraints appropriate for quantization error (e.g. $[0, 255]$ indexed image):

$$\|\mathcal{A}(X) - b\|_\infty \leq \varepsilon$$

# Example of RPCA

Background subtraction

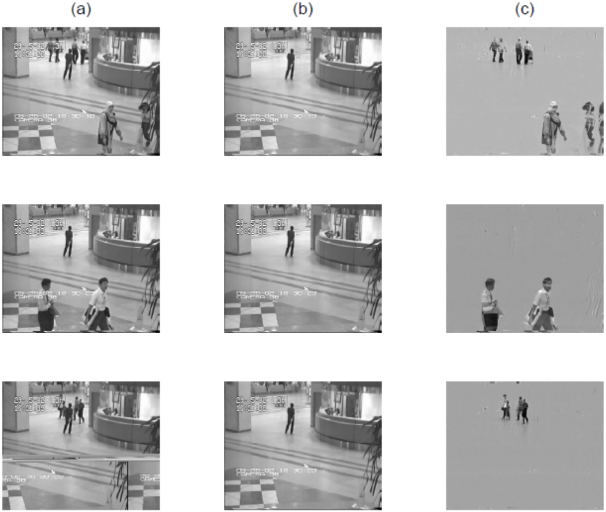


image from Goldfarb, Ma, Sheinberg '10

# Typical problems: sparse covariance selection

$\hat{\Sigma}$ is sample covariance matrix, $x \sim \mathcal{N}(0, \Sigma)$.

$(\Sigma^{-1})_{i,j} = 0 \implies x_i, x_j$ conditionally independent

**Sparse Covariance Selection**

$$X^\star = \underset{X}{\operatorname{argmin}} \; -\log\det X + \langle \hat{\Sigma}, X \rangle + \rho \|X\|_{\ell_1} \quad \text{such that} \quad X \succeq 0$$

If $\rho = 0$, then $X^\star = \Sigma^{-1}$.

Fancier: adding latent variables (Chandrasekaran, Parrilo, Willsky), which adds new constraint and trace norm.

# Existing approaches for RPCA

IPM: too slow (cf. Defeng Sun's talk)

First-order methods for RPCA (all 2008 – 2011 )

| Method | Name of code (Authors) |
|---|---|
| ADMM | LRSD (Yuan, Yang) |
| Non-convex ADMM | LMaFit (Shen, Wen, Zhang) |
| Fast ADMM | (Goldfarb, Ma, Sheinberg) |
| ADMM | IALM, Perceptions Lab at UIUC (Lin, Chen, Ma) |
| GP on dual | (Lin, Ganesh, Wright, Wu, Chen, Ma) |
| AGP on primal | Lin, Ganesh, Wright, Wu, Chen, Ma) |

# Augmented Lagrangian and variants

$$\min_{L,S} \|L\|_* + \lambda \|S\|_1 \quad \text{such that} \quad L + S = X, \mathcal{A}(X) = b \qquad \text{(RPCA)}$$

Augmented Lagrangian Method (ALM), aka Method of Multipliers (MM). Very simple when no inequality constraints.

$$\mathcal{L}_\mu(L,S,y) = \|L\|_* + \lambda\|S\|_1 + \langle y, \mathcal{A}(L+S) - b\rangle + \frac{\mu}{2}\|\mathcal{A}(L+S)-b\|_2^2$$

Alternating Direction Method

aka Alternating Direction Method of Multipliers (ADMM)

aka inexact ALM.

In ALM, primal update is too difficult due to coupling:

$$(L_k, S_k) = \operatorname{argmin} \mathcal{L}_\mu(L,S,y_{k-1})$$

So relax... (one-step of Jacobi method)

$$L_k = \operatorname{argmin} \mathcal{L}_\mu(L,S_{k-1},y_{k-1})$$

$$S_k = \operatorname{argmin} \mathcal{L}_\mu(L_{k-1},S,y_{k-1})$$

ADMM with constraints, fancier terms? Perhaps; ask Don Goldfarb.

# Challenges

1. Keep iterates low-rank when possible
2. Exploit sparsity
3. Allow constraints
4. Non-smooth, so *slower* convergence
5. How to project onto $\|Ax - b\|_2 \leq \varepsilon$ ?
6. Flexible

# TFOCS main idea

$$\min_x f(x) + \psi(\bar{A}x + \bar{b})$$

1. Find conic formulation*
2. Add strongly convex term
   - $f_\mu(x) = f(x) + \frac{\mu}{2}\|x - x_0\|^2$
   - can now calculate dual
   - dual is smooth
3. Solve dual problem
   - composite approach
   - $g = g_{\text{smooth}} + h$
   - $h$ nonsmooth but "nice"

Extends (e.g., atomic norms)

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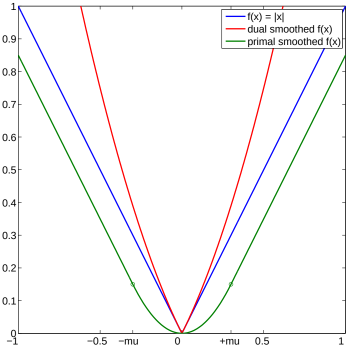
Extends (e.g., atomic norms)

Potential drawbacks:

- Primal iterate is not feasible
  - $\|Ax - b\| \le \varepsilon$, but $\varepsilon$ is estimate!
- Effect of smoothing
  - use continuation
  - made rigorous in proximal point framework
  - accelerated continuation
  - sometimes no effect even for $\mu > 0$

# Benefits of duality

1. Projection onto dual cone has no linear $\mathcal{A}$ term
2. Better smoothing: primal retains its kink




$$f^*(\lambda) \equiv \sup_x \langle \lambda, x \rangle - f(x)$$

$f$ strongly convex $\implies$ $f^*$ differentiable and $\nabla f^*$ Lipschitz

Smooth problems: *much* faster convergence, i.e. $\mathcal{O}(\frac{1}{k^2})$ vs $\mathcal{O}(\frac{1}{\sqrt{k}})$

# Example: matrix completion

$$\begin{array}{ll} \text{minimize} & \|X\|_{\mathsf{tr}} \\ \text{subject to} & \|\mathcal{A}(X) - b\| \le \varepsilon \end{array} \implies \begin{array}{ll} \text{minimize} & \|X\|_1 + \frac{\mu}{2}\|X - X_0\|_F^2 \\ \text{subject to} & (\mathcal{A}(X) - b, \varepsilon) \in \mathcal{K} \end{array}$$

Dual problem

$$\underset{\lambda}{\text{maximize}} \quad \underbrace{\inf_X \{\|X\|_{\mathsf{tr}} + \frac{\mu}{2}\|X - X_0\|^2 - \langle \lambda, \mathcal{A}(X) - b\rangle\}}_{-g_{\mathsf{smooth}}(\lambda)} - \underbrace{\varepsilon\|\lambda\|_*}_{h(\lambda)}$$

"Simple gradient" ($X_\lambda$ unique minimizer above)

# Example: matrix completion, version 2

$$\begin{array}{ll} \text{minimize} & \|X\|_{\text{tr}} \\ \text{subject to} & \|\mathcal{A}(X) - b\| \le \varepsilon \end{array} \quad \Longrightarrow \quad \begin{array}{ll} \text{minimize} & t + \frac{\mu}{2}\|X - X_0\|_F^2 \\ \text{subject to} & (\mathcal{A}(X) - b, \varepsilon) \in \mathcal{K} \\ & (X, t) \in \mathcal{K}_{\text{tr}} \end{array}$$

Dual problem

$$\underset{\lambda, (\nu, s) \in \mathcal{K}_{\text{spectral}}}{\text{maximize}} \quad \underbrace{-\varepsilon\|\lambda\|_*}_{h(\lambda)} + \ldots$$

$$\underbrace{\inf_{X, t} \{ t + \frac{\mu}{2}\|X - X_0\|_F^2 - \langle \lambda, \mathcal{A}(X) - b \rangle - \langle \nu, X \rangle - st \}}_{-g_{\text{smooth}}(\lambda)}$$

Similar algorithm, but now $X_{\lambda,\nu}$ is linear in $\lambda$ and $\nu$, so dual is constrained quadratic (and with $2\times$ variables).

# General form

Exploit structure, not just "black-box"
Two viewpoints: conic dual or Fenchel dual

Fenchel duality view

$$\min f(x) + \sum_i \psi_i(A_i x - b_i)$$

where $f, \psi_i^*$ are "prox-capable", $\psi_i \to \bar{\mathbb{R}}$

$$\text{prox}_f(y) = \operatorname*{argmin}_x f(x) + \frac{1}{2}\|x - y\|^2$$

Matrix completion: $\psi_1(X) = \iota_{\{X : \|X\| \le \varepsilon\}}$, $A_1 = \mathcal{A}$, $b_1 = b$.

1. matrix completion style 1 corresponds to:
   $f(x) = \|X\|_1, \quad \psi_2 = 0$
2. matrix completion style 2 corresponds to:
   $f = 0, \quad \psi_2(x) = \|X\|_{\text{tr}}, A_2 = I, b_2 = 0$

If $f = 0$, dual is always (constrained) quadratic.

# Solving the dual

"Proximal gradient descent", aka "forward-backward" algorithm. Handles smooth + nonsmooth (Fukushima and Mine, 1981).

- Gradient projection step for minimizing smooth $g$:

$$\lambda_{k+1} \leftarrow \underset{\lambda \in \mathcal{K}^*}{\operatorname{argmin}}\ g(\lambda_k) + \langle \nabla g(\lambda_k), \lambda - \lambda_k \rangle + \frac{L}{2} \|\lambda - \lambda_k\|^2$$

- Generalized gradient projection for minimizing $g + h$ ($h$ nonsmooth)

$$\lambda_{k+1} \leftarrow \underset{\lambda}{\operatorname{argmin}}\ g(\lambda_k) + \langle \nabla g(\lambda_k), \lambda - \lambda_k \rangle + \frac{L}{2} \|\lambda - \lambda_k\|^2 + h(\lambda)$$

- Solution is proximity operator of $h$. Often known.
  - Ex. $h = \chi_C$, then proximity operator is just projection onto $C$
  - Ex. $h = \|x\|_1$, then proximity operator is shrinkage
- Works with backtracking and Nesterov acceleration
- Popularized in 2005: Nesterov, Beck, Teboulle

# Generic algorithm (Nesterov's style)

Auslender-Teboulle version, no backtracking

$\min_x f(x) + \psi(\bar{\mathcal{A}}x + \bar{b}), \quad h \stackrel{\text{def}}{=} \psi^*$

**Algorithm 1** Generic algorithm for the conic standard form

**Require:** $\lambda_0, x_0 \in \mathbb{R}^n$, $\mu > 0$, step sizes $\{t_k\}$

$\theta_0 \leftarrow 1,\ v_0 \leftarrow \lambda_0$

**for** $k = 0, 1, 2, \ldots$ **do**

$\quad \nu_k \leftarrow (1-\theta_k) v_k + \theta_k \lambda_k$

$\quad x_k \leftarrow \operatorname{argmin}_x f(x) + \mu/2 \|x - x_0\|^2 - \langle \bar{\mathcal{A}}^T(\nu_k), x \rangle$

$\quad \lambda_{k+1} \leftarrow \operatorname{argmin}_\lambda h(\lambda) + \frac{\theta_k}{2t_k} \|\lambda - \lambda_k\|^2 + \langle \bar{\mathcal{A}}(x_k) + \bar{b}, \lambda \rangle$

$\quad v_{k+1} \leftarrow (1-\theta_k) v_k + \theta_k \lambda_{k+1}$

$\quad \theta_{k+1} \leftarrow 2/(1 + (1 + 4/\theta_k^2)^{1/2})$

**end for**

$x$ is primal

$\lambda, \nu, v$ are dual, $\theta$ is scalar

# Algorithm for Matrix Completion

Matrix completion, style 1

**Algorithm 2** Algorithm for nuclear-norm minimization ($\ell_2$ constraint)

4: $X_k \leftarrow \mathrm{SoftThresholdSingVal}(X_0 - \mu^{-1}\mathcal{A}^T(\nu_k), \mu^{-1})$
5: $\lambda_{k+1} \leftarrow \mathrm{Shrink}(\lambda_k - \theta_k^{-1} t_k (b - \mathcal{A}(X_k)), \theta_k^{-1} t_k \epsilon)$

$$\mathrm{SoftThreshold}(x,\tau) = \mathrm{sgn}(x) \cdot \max\{|x| - \tau, 0\}$$

$$\mathrm{SoftThresholdSingVal}(X,t) = U \cdot \mathrm{SoftThreshold}(\Sigma, t) \cdot V^T,$$

$$\mathrm{Shrink}(z,t) \triangleq \max\{1 - t/\|z\|_2, 0\} \cdot z = \begin{cases} 0, & \|z\|_2 \le t, \\ (1 - t/\|z\|_2) \cdot z, & \|z\|_2 > t. \end{cases}$$

Significantly extends SVT

# Other new algorithms

**Algorithm 3** Algorithm excerpt for Dantzig

4: $x_k \leftarrow \text{SoftThreshold}(x_0 - \mu^{-1} A^T A \nu_k, \mu^{-1})$.
5: $\lambda_{k+1} \leftarrow \text{SoftThreshold}(\lambda_k - \theta_k^{-1} t_k A^T (b - A x_k), \theta_k^{-1} t_k \delta)$

**Algorithm 4** Algorithm excerpt for LASSO

4: $x_k \leftarrow \text{SoftThreshold}(x_0 - \mu^{-1} A^T \nu_k, \mu^{-1})$
5: $\lambda_{k+1} \leftarrow \text{Shrink}(\lambda_k - \theta_k^{-1} t_k (b - A x_k), \theta_k^{-1} t_k \epsilon)$

**Algorithm 5** Algorithm excerpt for TV minimization

4: $x_k \leftarrow x_0 + \mu^{-1}(\Re(D^* \nu_k^{(1)}) - A^* \nu_k^{(2)})$
5: $\lambda_{k+1}^{(1)} \leftarrow \text{CTrunc}(\lambda_k^{(1)} - \theta_k^{-1} t_k^{(1)} D x_k, \theta_k^{-1} t_k^{(1)})$
$\lambda_{k+1}^{(2)} \leftarrow \text{Shrink}(\lambda_k^{(2)} - \theta_k^{-1} t_k^{(2)} (b - A x_k), \theta_k^{-1} t_k^{(2)} \epsilon)$

# Conic Programs

$$\min_x \langle c, x \rangle \quad \text{such that} \quad x \geq_{\mathcal{K}} 0,\ Ax = b$$

$$\begin{array}{lcl} \mathcal{K} = \mathbb{R}^n_+ & \Longrightarrow & \text{LP} \\ \mathcal{K} = \{(x,t) \in \mathbb{R}^{n+1} : ||x||_2 \leq t\} & \Longrightarrow & \text{SOCP} \\ \mathcal{K} = S^n_+ & \Longrightarrow & \text{SDP} \end{array}$$

Dual, before smoothing

$$\max_{\nu,\lambda} -\langle b, \nu \rangle \quad \text{such that} \quad \lambda \geq_{\mathcal{K}^*} 0, \quad \lambda = c + A^* \nu$$

Dual, after smoothing

$$\max_{\nu,\lambda} -\langle b, \nu \rangle - \frac{1}{2\mu} \|c - \lambda + A^* \nu\|^2 + \langle c - \lambda + A^* \nu, x_0 \rangle \quad \text{such that} \quad \lambda \geq_{\mathcal{K}^*} 0.$$

# Related work

Inspiration: apply SVT 2008 (aka linearized Bregman 2008, aka Uzawa) to Dantzig.

A good idea should be discovered many times! Much recent work in similar flavors:

- O. Mangasarian 1981. Add quadratic term to objective of linear program and solve dual.
- PPPA, F. Malgouyres and T. Zeng 2008. Use continuation, prove convergence of $Ax$, not $x$. Conclude line search is not worth it.
- Y.-J. Liu, D. Sun, K.C. Toh 2009. Similar approach: apply proximal point algorithm (not accelerated), solve inner problems with APG (like TFOCS). For matrix completion problem.
- Chambolle and Pock 2010, unconstrained versions, for general functions
- Combettes and Pesquet 2009, discuss forward-backward applied to dual of smoothed problem, for general functions.
- Combettes, Dũng and Vũ 2010, extend previous work and prove convergence (no rate).

# TFOCS ideas: extras

Software is modular. Easy to add constraints, change solver...

## (Important) details

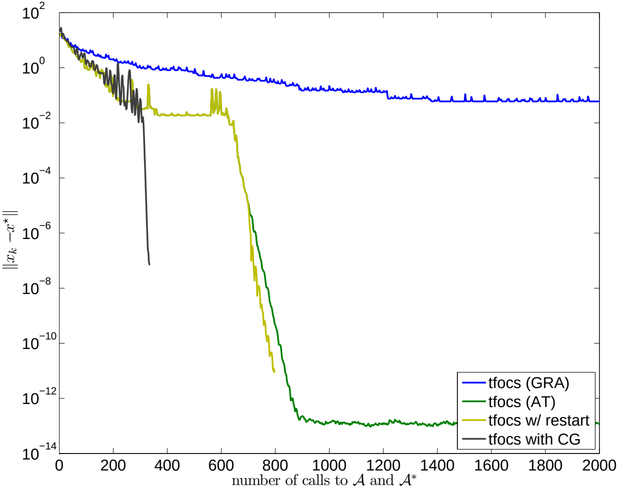
- 6 first-order methods (GRA + 5 accelerated methods)
- Efficient step size procedures (based on Tseng's convergence analysis): no Lipschitz constant needed; like Gonzaga/Karas/Rossetto. Key idea: if $L$ updated, $\theta$ must be updated as well
- Easy testing and benchmarking
- Efficient use of linear operator structure: crucial when backtracking occurs

$$\text{minimize} \quad g_{\text{smooth}}(\mathcal{A}^T\lambda) + h(\lambda)$$

- Accelerated continuation: remove effect of $\mu$
- Exact perturbation
- Restart strategies
- Convergence proofs

# Conjugate Gradient

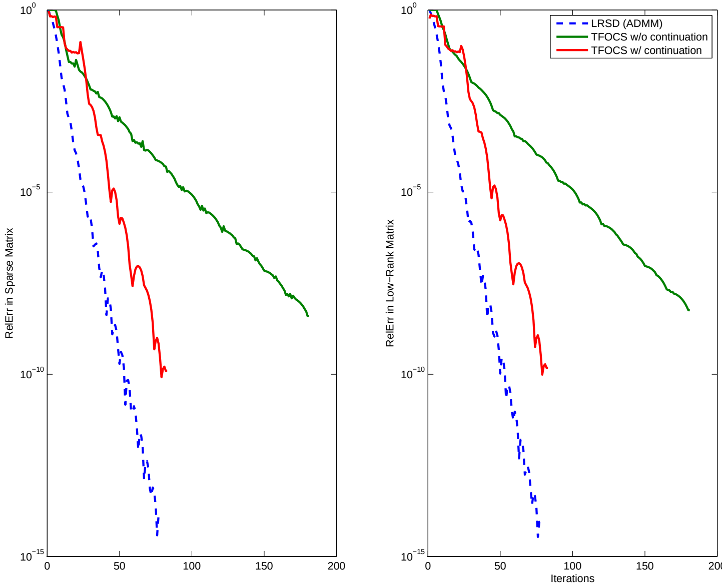
Advantage of modularity: easy to try new solvers, line search.
Plans for non-linear CG, (L-)BFGS, SESOP, Karimi/Vavasis ...




Ex: Non-linear CG (Polak-Ribiere), noiseless basis pursuit, $N = 2048$.

# Comparison on RPCA

Compare with LRSD (ADMM code by Yuan, Yang)



# Standard continuation

Want perturbation small

$$\begin{array}{ll} \text{minimize} & f(x) + \frac{1}{2}\mu\|x - x_0\|^2 \\ \text{subject to} & \mathcal{A}(x) + b \in \mathcal{K} \end{array}$$

Problem: $L \propto 1/\mu$

**Algorithm 6** Standard continuation

**Require:** $Y_0$, $\mu_0 > 0$, $\beta < 1$

1: **for** $j = 0, 1, 2, \ldots$ **do**

2: $X_{j+1} \leftarrow \underset{\mathcal{A}(x)+b\in\mathcal{K}}{\operatorname{argmin}} f(x) + \frac{\mu_j}{2}\|x - Y_j\|_2^2$

3: $Y_{j+1} \leftarrow X_{j+1}$ (or $Y_{j+1} \leftarrow Y_0$)

4: $\mu_{j+1} \leftarrow \beta\mu_j$

5: **end for**

FPC: Hale, Yin, and Zhang ('08)

# Moreau-Yosida regularization

Moreau envelope $$h(Y) = \min_{x \in C} f(x) + \frac{\mu}{2}\|x - Y\|_2^2$$

Moreau proximity operator $$X_Y = \operatorname*{argmin}_{x \in C} f(x) + \frac{\mu}{2}\|x - Y\|_2^2$$

## Theorem

*$h$ is continuously differentiable with gradient*

$$\nabla h(Y) = \mu(Y - X_Y)$$

*The gradient is Lipschitz continuous with constant $L = \mu$*

Minimizing $h$ by gradient descent $\rightarrow$ proximal point algorithm (PPA) (Rockafellar, 70s)

# Accelerated continuation (Nesterov style)

If proximal-point algorithm is gradient descent, then why not accelerate?

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**Algorithm 7** Accelerated continuation

---

**Require:** $Y_0$, $\mu_0 > 0$

1: $X_0 \leftarrow Y_0$

2: **for** $j = 0, 1, 2, \ldots$ **do**

3: $\quad X_{j+1} \leftarrow \underset{\mathcal{A}(x)+b\in\mathcal{K}}{\operatorname{argmin}} f(x) + \frac{\mu_j}{2}\|x - Y_j\|_2^2$

4: $\quad Y_{j+1} \leftarrow X_{j+1} + \frac{j}{j+3}(X_{j+1} - X_j)$

5: $\quad$ (optional) increase or decrease $\mu_j$

6: **end for**

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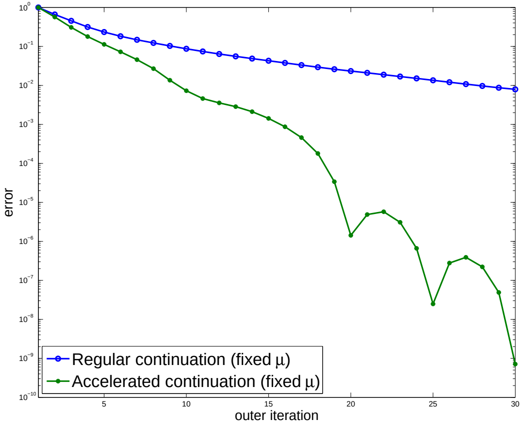
Keep $\mu_j \equiv \mu$ so subproblems quick to solve

Warm-start subproblems

For small $\mu$, typically 5 iterations

# Simple vs. accelerated continuation: LASSO example




$\|x_k - x^\star\| / \|x_0 - x^\star\|$ vs. outer iteration count

# Effect of perturbation

Nice surprise:

Linear programs (ex. Dantzig, Basis Pursuit) have exact penalty

## Theorem (Exact penalty)

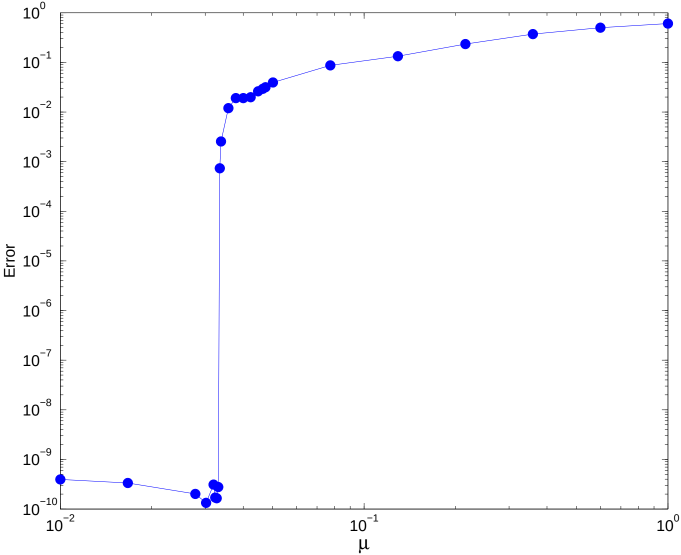
- *Arbitrary LP with objective* $\langle c, x\rangle$ *and with opt. solution*
- *Perturbed LP with objective* $\langle c, x\rangle + \frac{1}{2}\mu\|x - x_0\|_Q^2$, $Q \succeq 0$

*There is* $\mu_0 > 0$ *s.t. for* $0 < \mu \leq \mu_0$*, any solution to perturbed problem is a solution to LP*

- Special case (BP): Yin ('10)
- More general result: Friedlander and Tseng ('07)
- Combine with continuation $\implies$ finite termination
  Known since Bertsekas '75, Polyak and Tretjakov '74, Mangasarian '79

# Illustration

Exact penalty for Dantzig Selector (since linear program)



# Parameters

## Lipschitz Gradient

$$f(y) \le f(x) + \langle y - x, \nabla f(x) \rangle + \frac{L}{2} \|x - y\|_2^2$$

## Strong Convexity

$$f(y) \ge f(x) + \langle y - x, \nabla f(x) \rangle + \frac{m_f}{2} \|x - y\|_2^2$$

If $\nabla^2 f$ exists, equivalent to

$$m_f I \preceq \nabla^2 f \preceq LI$$

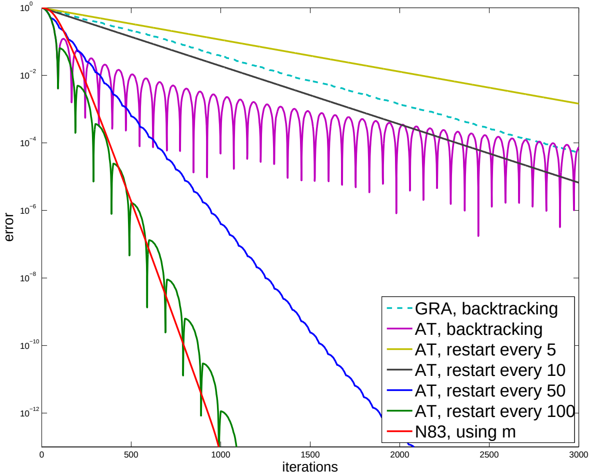
Goal: user needs no knowledge of $m_f$ and $L$

- For $L$, trick: backtracking line search
- For $m_f$, trick: restart

# Restart

Problem: accelerated schemes don't automatically take advantage of strong convexity.

i.e. $m_f$ unknown $\implies$ no linear convergence



# Restart

Convergence of accelerated method:

$$f(x_k) - f^\star \le \frac{L}{k^2}\|x^\star - x_0\|^2$$

If $f$ is strongly convex with parameter $m_f$,

$$\|x_k - x^\star\|^2 \le \frac{2L}{m_f}\frac{1}{k^2}\|x^\star - x_0\|^2$$

With restart, $x_0$ is $x_k$ of a previous sequence. Do this $j$ times.

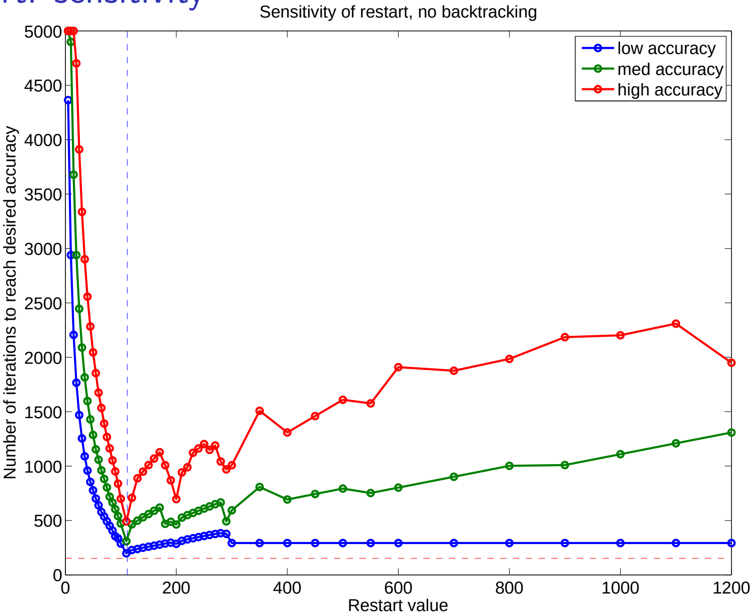
$$\|x_{jk} - x^\star\| \le \left(\sqrt{\frac{2L}{m_f}}\frac{1}{k}\right)^j \|x^\star - \hat{x}_0\|$$

This is linear convergence with rate $\rho = \left(\sqrt{\frac{2L}{m_f}}\frac{1}{k}\right)^{1/k}$.

$$k_{\text{opt}} = e\sqrt{\frac{2L}{m_f}}$$

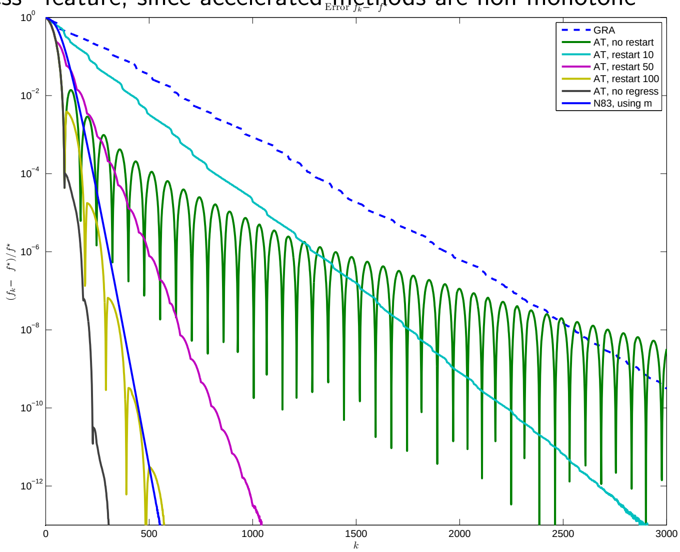
See PARNES paper (Gu, Lim, Wu 2009), Nesterov 2007, and also Nemirovskii-Yudin (80s).
Goes back to Powell (1977) for non-linear CG.

# Restart: sensitivity



# Restart: improvements

"No Regress" feature, since accelerated methods are non-monotone



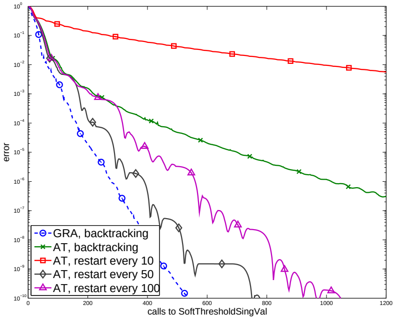
# Special considerations for matrix completion

(1) Keep $\mu$ small always, to keep $X_k$ low-rank:

$$X_k \leftarrow \text{SoftThresholdSingVal}(X_0 - \mu^{-1}\mathcal{A}^T(\nu_k), \mu^{-1})$$

(2) Gradient descent performs well! Why?

- automatically exploits strong convexity
- More robust to errors in SVD calculation (cf. Devolder/Nesterov )



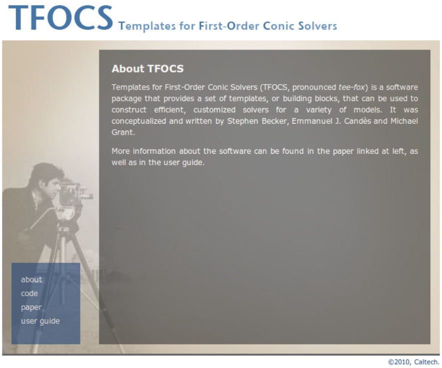

Note: advantage of gradient descent only appears with equality constraints

# Convergence rates

- Inner iterations: objective converges in $\mathcal{O}(1/k^2)$
- Outer iterations: if via proximal point method, locally linear, or globally $\mathcal{O}(1/j)$. If via accelerated proximal point method, $\mathcal{O}(1/j^2)$.
- How to combine the two? One method: Liu/Sun/Toh 2009
- Or, result of Güler 1990s, on inexact accelerated proximal point method. Need primal variables of inner iterates to converge.
- Key result: Fadili/Peyré March 2011.
- Preliminary work: $\mathcal{O}(\varepsilon^{-5/4})$ iterations to reach $\varepsilon$-solution

# Software release

- Paper
- User guide
- Software (MATLAB)
    - solvers
    - many simple examples
    - a few real-world examples
    - continuation wrappers
    - compatible with SPOT
- Parameters: any $\mu > 0$




http://tfocs.stanford.edu

# Example in TFOCS

## Basis Pursuit Denoising $BP_\varepsilon$, analysis

$$\min_x \|Wx\|_1 \quad \text{such that} \quad \|Ax-b\|_2 \le \varepsilon$$

```
prox   = { prox_l2( epsilon ), proj_linf };
linear = { A, -b; W, 0 };
x      = tfocs_SCD( [], linear, prox, mu, x0 };
```

Easy to add constraints, e.g. $x \ge 0$

```
prox  = { prox_l2( epsilon ), proj_linf, proj_Rplus };
linear = { A, -b; W, 0; 1, 0 };
```

Of course, this is also builtin. . .

```
x    = solver_sBPDN_W(A,W,b,epsilon,mu)
```

*No Lipschitz constant or step size needed!*