5.3 #6 #9 #15 #21 #24 #49

#6

$$\begin{bmatrix} 3 & 2 \\ -2 & -3 \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 3-\lambda & 2 \\ -2 & -3-\lambda \end{vmatrix}$$

$$= -(3-\lambda)(3+\lambda) + 4$$

$$= -(9-\lambda^2) + 4 = \lambda^2 - 9 + 4 = \lambda^2 - 5$$

$$\lambda = \pm\sqrt{5}.$$

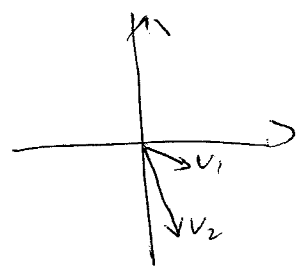
$\lambda = \sqrt{5}$.

$$\begin{bmatrix} 3 & 2 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \sqrt{5}\begin{bmatrix} x \\ y \end{bmatrix}$$

$\Rightarrow$ $V_1 = [1, \frac{1}{2}\sqrt{5} - \frac{3}{2}]$

$\lambda = -\sqrt{5}$

$V_2 = [1, -\frac{1}{2}\sqrt{5} - \frac{3}{2}]$



$\boxed{\#9}$ $\begin{bmatrix} 1 & 4 \\ -4 & 11 \end{bmatrix}$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 4 \\ -4 & 11-\lambda \end{vmatrix} = \lambda^2 - 12\lambda + 27 = (\lambda-3)(\lambda-9) = 0$$

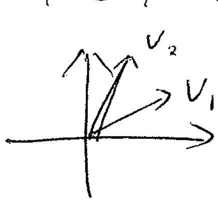
$\lambda_1 = 3 \quad \lambda_2 = 9$

$\lambda_1 = 3$

$$\left|\begin{array}{cc|c} -2 & 4 & 0 \\ -4 & 8 & 0 \end{array}\right| \rightarrow \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right], \quad V_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\lambda_2 = 9$

$$\left|\begin{array}{cc|c} -8 & 4 & 0 \\ -4 & 2 & 0 \end{array}\right| \rightarrow \begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad V_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



#15 $\begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}$

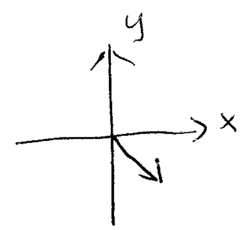
$$|A-\lambda I| = \begin{vmatrix} 2-\lambda & -1 \\ 1 & 4-\lambda \end{vmatrix}$$

$$\lambda^2+6\lambda+9=0 \quad \lambda = 3\ 3$$

$\lambda = 3$

$$\left[\begin{array}{cc|c} -1 & -1 & 0 \\ 1 & 1 & 0 \end{array}\right] \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



$\boxed{\#21}$ $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$

$$|A-\lambda I| = \begin{vmatrix} 1-\lambda & 2 & 0 \\ 2 & -\lambda & 3 \\ 2 & 3 & -\lambda \end{vmatrix} = \lambda^3-\lambda^2-17\lambda-15=0$$

$$\lambda_1 = 5 \quad \lambda_2 = -3 \quad \lambda_3 = -1.$$

$\lambda_1 = 5$.

$$\left[\begin{array}{ccc|c} -4 & 2 & 0 & 0 \\ 2 & -5 & 3 & 0 \\ 2 & 3 & -5 & 0 \end{array}\right] \Rightarrow \quad v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$\lambda_2 = -3$.

$$v_2 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$v_3 = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$$

Each eigenvalue corresponds to a one-dimensional eigenspace.

#24 $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 0 \\ 3 & 2 & -2 \end{bmatrix}$.

$$0 = |A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 \\ -1 & 3-\lambda & 0 \\ 3 & 2 & -2-\lambda \end{vmatrix} = (\lambda-1)(\lambda-3)(\lambda+2)$$

$\Rightarrow \lambda_1 = 1 \quad \lambda_2 = 3 \quad \lambda_3 = -2$.

If $\lambda_1 = 1$

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 3 & 2 & -3 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 3 & 2 & -3 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 3 & 2 & -3 & 0 \\ 0 & 8 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 12 & 0 & -9 & 0 \\ 0 & 8 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\boxed{V_1 = \begin{pmatrix} 6 \\ 3 \\ 8 \end{pmatrix}}$$

If $\lambda_2 = 3$ $\boxed{V_2 = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}}$

$\lambda_3 = -2$ $\boxed{V_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}$

Each eigenvalue correspond to a one-dim. space.

$\boxed{\#49}$ $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$.

both have eigenvalue $\lambda_1 = 1$ $\lambda_2 = 2$.

eigenvector of A. $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

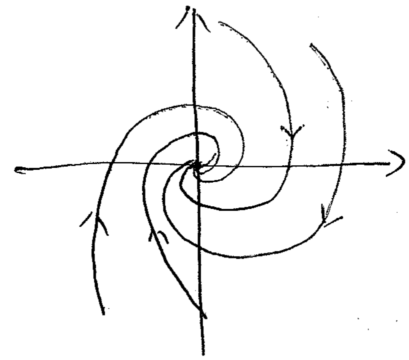
eigenvector of A^T $v_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

6.2 #1 #5 #6 #7 #8 #14 #17 #28 #41

#1. $x'' + x' + x = 0$.

(a). $x' = y$

$y' = -x - y$



(b) equilibrium point $(x, y) = (0, 0)$.

h - nullcline. $x + y = 0$

(c). v - nullcline. $y = 0$.

(d). $(x, y) = (0, 0)$ is stable.

(e). A mass-spring system with this equation. shows damped oscillatory motion

#5. A

$\boxed{\#6}$ C

#7 D

#8 B

#14. $X' = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} X$

$p(\lambda) = \begin{vmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{vmatrix} = \lambda^2 - 4\lambda - 5 = 0.$

$\lambda_1 = -1 \quad \lambda_2 = 5.$

If $\lambda_1 = -1 \Rightarrow v_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\lambda_2 = 5 \Rightarrow v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\Rightarrow x(t) = c_1 e^{-t} \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_2 e^{5t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$

#17. $x' = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} x$

$$p(\lambda) = \begin{vmatrix} 3-\lambda & -2 \\ 2 & -2-\lambda \end{vmatrix} = \lambda^2 - \lambda - 2 = 0$$

$\Rightarrow \lambda_1 = -1 \quad \lambda_2 = 2$

If $\lambda_1 = -1 \Rightarrow v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\lambda_2 = 2 \Rightarrow v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$$x(t) = c_1 e^{-t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$\boxed{\#28}$ $x' = \begin{pmatrix} -2 & 4 \\ 1 & 1 \end{pmatrix} x \qquad x(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$$p(\lambda) = \begin{vmatrix} -2-\lambda & 4 \\ 1 & 1-\lambda \end{vmatrix} = \lambda^2 + \lambda - 6 = 0$$

$\Rightarrow \lambda_1 = -3 \quad \lambda_2 = 2$.

If $\lambda_1 = -3 \Rightarrow v_1 = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$

$\lambda_2 = 2 \Rightarrow v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\Rightarrow x(t) = c_1 e^{-3t} \begin{pmatrix} -4 \\ 1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$x(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$$\Rightarrow \quad \begin{aligned} -4c_1 + c_2 &= -1 \\ c_1 + c_2 &= 1 \end{aligned}$$

$\Rightarrow c_1 = \frac{2}{5} \quad c_2 = \frac{3}{5}$

$$\Rightarrow x(t) = \left(\frac{2}{5}\right) e^{-3t} \begin{pmatrix} -4 \\ 1 \end{pmatrix} + \frac{3}{5} e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

#41 $X' = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{pmatrix} X$

$$P(\lambda) = \begin{vmatrix} -1-\lambda & 1 & 0 \\ 1 & 2-\lambda & 1 \\ 0 & 3 & -1-\lambda \end{vmatrix} = -\lambda^3 + 7\lambda + 6 = 0$$

$\Rightarrow \lambda_1 = -1,\ \lambda_2 = 3\ \ \lambda_3 = -2$

$\lambda_1 = -1 \Rightarrow v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$\lambda_2 = 3 \Rightarrow v_2 = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$

$\lambda_3 = -2 \Rightarrow v_3 = \begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix}$

$$\Rightarrow x(t) = C_1 e^{-t}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + C_2 e^{3t}\begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} + C_3 e^{-2t}\begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix}$$