6.3 10, 16, 36

10

$$\vec{x}' = \begin{bmatrix} -2 & -3 \\ 3 & -2 \end{bmatrix} \vec{x}$$

$$\lambda = -2 \pm 3i$$

$$\vec{v} = \begin{bmatrix} \pm i \\ 1 \end{bmatrix}$$

$$\alpha = -2, \quad \beta = 3$$

$$P = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad Q = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\vec{x}_1(t) = e^{\alpha t} \cos\beta t \vec{P} - e^{\alpha t} \sin\beta t \vec{Q}$$

$$\vec{x}_2(t) = e^{\alpha t} \sin\beta t \vec{P} + e^{\alpha t} \cos\beta t \vec{Q}$$

$$\vec{x}(t) = c_1 \vec{x}_1(t) + c_2 \vec{x}_2(t)$$

$$\boxed{\vec{x}(t) = c_1 e^{-2t} \begin{bmatrix} -\sin 3t \\ \cos(3t) \end{bmatrix} + c_2 e^{-2t} \begin{bmatrix} \cos(3t) \\ \sin(3t) \end{bmatrix}}$$

16

$$\vec{x}' = \begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix} \vec{x} \qquad x(0) = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$\lambda = -1 \pm i$$

$$\vec{v} = \begin{bmatrix} 5 \\ 2 \mp i \end{bmatrix} \qquad \alpha = -1 \quad \beta = 1$$

$$\vec{x}_1 = e^{\alpha t} \cos\beta t\, P - e^{\alpha t} \sin\beta t\, Q$$

$$\vec{x}_2 = e^{\alpha t} \sin\beta t\, P + e^{\alpha t} \cos\beta t\, Q$$

$$\vec{x}(t) = c_1 x_1 + c_2 x_2$$

$$x(0) = \begin{bmatrix} 5 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$x(t) = e^{-t} \begin{bmatrix} 5\cos t - 10\sin t \\ -5\sin t \end{bmatrix}$$

36

$$\vec{x}' = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \vec{x}, \quad \vec{x}(0) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda_1 = 0 \qquad \lambda_2, \lambda_3 = \pm i\sqrt{2}$$

$$\vec{v}_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \qquad \vec{v}_{2,3} = \begin{bmatrix} 1 \\ \pm i\sqrt{2} \\ 1 \end{bmatrix}$$

$$\vec{x}_1(t) = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\vec{x}_2(t) = \cos\sqrt{2}t \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \sqrt{2}\sin\sqrt{2}t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{x}_3(t) = \sin\sqrt{2}t \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \sqrt{2}\cos\sqrt{2}t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{x}(0) = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \\ \frac{\sqrt{2}}{2} \end{bmatrix} \Longrightarrow$$

$$\vec{x}(t) = \frac{1}{2}\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} \cos\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t \\ \cos\sqrt{2}t \end{bmatrix} + \frac{\sqrt{2}}{2}\begin{bmatrix} \sin\sqrt{2}t \\ \sqrt{2}\cos\sqrt{2}t \\ \sin\sqrt{2}t \end{bmatrix}$$

6.4 3, 5, 17

[3]

$$\vec{x}' = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \vec{x}$$

$\lambda = -2, -2$

$\vec{V}_1 \neq \vec{V}_2$

ASYMPTOTICALLY STABLE

STAR NODE

[5] $\vec{x}' = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \vec{x}$

$\lambda = 1, 5$

UNSTABLE NODE

17

$$\vec{x}' = \begin{bmatrix} k & 0 \\ 0 & -1 \end{bmatrix} \vec{x}$$

$$p(\lambda) = (\lambda - k)(\lambda + 1) = 0$$

$$\lambda = k, -1$$

(a) $k \in (-\infty, 1) \Rightarrow (0,0)$ ASYMPTOTICALLY STABLE

NON DEGENERATE NODE

(b) $k = -1 \Rightarrow (0,0)$ ASYMPTOTICALLY STABLE

STAR NODE

(c) $k \in (-1, 0) \Rightarrow (0,0)$ ASYMPTOTICALLY STABLE

NON DEGENERATE NODE

(d) $k = 0 \Rightarrow$ A singular

x_1-axis EQUILIBRIUM (STABLE)

(e) $k \in (0, \infty) \Rightarrow (0,0)$ UNSTABLE SADDLE

POINT