Sect. 1.2:

8.) Soln: $y = e^t \cos t + ce^t \Rightarrow y' = e^t \cos t - e^t \sin t + ce^t$

DE: $y' - y = -e^t \sin t$

Subbing in to the DE:

$$e^t \cos t - e^t \sin t + ce^t - (e^t \cos t + ce^t) \overset{?}{=} -e^t \sin t$$

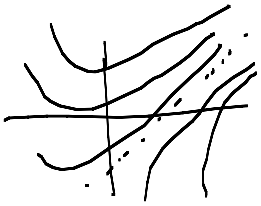
$$\cancel{e^t \cos t} - e^t \sin t + \cancel{ce^t} - \cancel{e^t \cos t} - \cancel{ce^t}$$

$$= -e^t \sin t$$

$y(0) = -1 \Rightarrow \quad -1 = e^0 \cos 0 + ce^0 = -1 = 1 + c$

$\Rightarrow c = -2$

11.)



Looks exponential and asymptotic to a diagonal line.

$y' + y = t \Rightarrow$ int. factor: e^t.

$$e^t y' + e^t y = te^t \Rightarrow \frac{d}{dt}(e^t y) = te^t \Rightarrow e^t y = \int te^t dt = te^t - e^t + c$$

$u = t \quad dv = e^t dt$

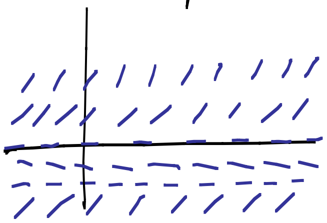
$du = dt \quad v = e^t$

$\therefore y = t - 1 + ce^{-t}$

$y' = 1 - ce^{-t} \Rightarrow \cancel{1} \cancel{- ce^{-t}} + t \cancel{- 1} + \cancel{ce^{-t}} = t$

14.) $y' = y(y+1)$

direction field: plot some points



y	y'
0	0
1	2
-1	0
$-\frac{1}{2}$	$-\frac{1}{4}$
-2	2
2	6

$y' = 0 \Rightarrow y(y+1) = 0 \Rightarrow y = 0, -1$ are equilibria

stable : -1

unstable : 0

16.) $y' = 1$ ⓒ

all 1

17.) $y' = y$ ⓓ

slope 1 at 1

slope 2 at 2

slope 0 at 0

same for all t

18.) $y' = y/t$ ⓕ

at $(1, y)$, slope is y

at $(0, y)$, ∞ slope

at $(-1, y)$ slope is $-y$

19.) $y' = t^2$ ⓑ

same for all y

$t = 0 \Rightarrow y' = 0.$

20.) $y' = t^2 + y^2$ ⓔ

$(0,0) \Rightarrow y' = 0$

$(1,1) \Rightarrow y' = 2$

slope is always ≥ 0

21.) $y' = \frac{1}{t}$ ⓐ

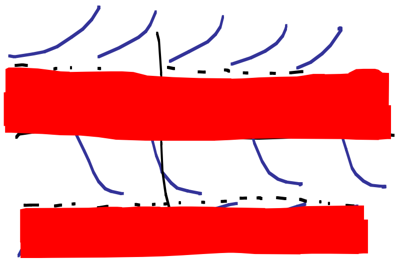
same for all y

$t = 0 \Rightarrow y' = \infty$

$t = 1 \Rightarrow y' = 1$

22.) $y' = y^2 - 4$

$$y'' = 2y\,y'$$
$$= 2y(y^2 - 4)$$
$$= 2y^3 - 8y$$



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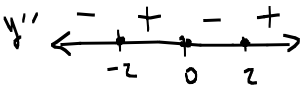
$$0 = 2y(y^2 - 4)$$

$y = 0, \pm 2$ ← all inflections



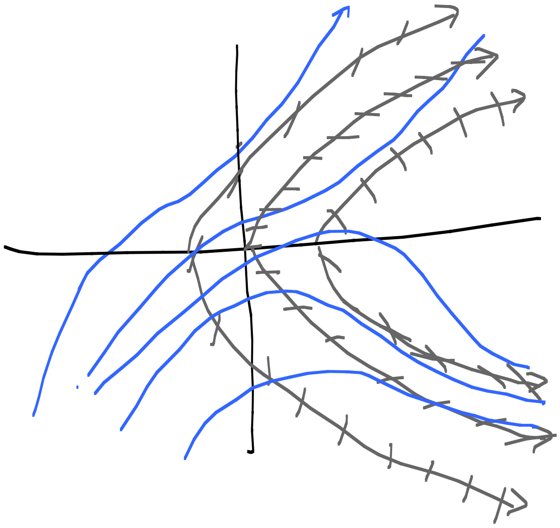
3b) $y' = y^2 - t$

$0 = y^2 - t \Rightarrow t = y^2$

$1 = y^2 - t \Rightarrow t = y^2 - 1$

$-1 = y^2 - t \Rightarrow t = y^2 + 1$

$2 = y^2 - t \Rightarrow t = y^2 - 2$



5b.) $y' = \frac{1}{ty}$

a. $0 = \frac{1}{ty}$ Nope.

b. Not defined when $t=0$ or $y=0$.

c. $a = \frac{1}{ty} \Leftrightarrow aty = 1 \Leftrightarrow y = \frac{1}{at}$. Nope.

d. $y' = \frac{1}{ty} \Rightarrow y'' = -\frac{1}{t^2y^2}(y + ty')$

$$= -\frac{y + t\frac{1}{ty}}{t^2y^2} = -\frac{y + \frac{1}{y}}{t^2y^2}$$

Concave down if $y + \frac{1}{y} > 0 \Leftrightarrow y > -\frac{1}{y}$

$\underline{y>0}$: $y^2 > -1$ $\forall y \neq 0$

$\underline{y<0}$: $y^2 < -1$ No solns.

Concave up if $y + \frac{1}{y} < 0 \Leftrightarrow y < -\frac{1}{y}$

$\underline{y>0}$: $y^2 < -1$ No solns

$\underline{y<0}$: $y^2 > -1$ $\forall y \neq 0$

Neither: $y = 0$

e. As $t \to \infty$, solns $\to \infty$ in upper ½ plane; $-\infty$ in lower

f. As $t \to -\infty$, solns come from ∞ in upper and $-\infty$ in lower

g. Vert. asymp at $t=0$ (or as $y \to 0$). No periodic

Sect. 1.3

7.) $y' = \frac{e^{t+y}}{y+1} = \frac{e^t e^y}{y+1} \Rightarrow \frac{(y+1)}{e^y} dy = e^t dt$ separable

$0 = \frac{e^t e^y}{y+1}$ Never (alt: $\frac{e^y}{y+1} = g(y) \neq 0$)

17.) $y' = y^2 - 4 \Rightarrow \frac{y'}{y^2-4} = 1 \Rightarrow \int \frac{1}{y^2-4} dy = \int 1 dt$

$$\frac{1}{(y+2)(y-2)} = \frac{A}{y-2} + \frac{B}{y+2}$$

$$\frac{1}{y+2} = A + \frac{B(y-2)}{y+2} \xrightarrow{y=2} \frac{1}{4} = A$$

$$\frac{1}{y-2} = \frac{A(y+2)}{y-2} + B \xrightarrow{y=-2} -\frac{1}{4} = B$$

$$\int \frac{1}{y^2-4} dy = \frac{1}{4}\int \frac{1}{y-2} dy - \frac{1}{4}\int \frac{1}{y+2} dy = \frac{1}{4}\ln|y-2| - \frac{1}{4}\ln|y+2|$$

So: $\frac{1}{4}\ln\left|\frac{y-2}{y+2}\right| = 1+c \Rightarrow \frac{y-2}{y+2} = Ae^{4t}$

$$y-2 = Ae^{4t}y + 2Ae^{4t} \Rightarrow y - Ae^{4t}y = 2 + 2Ae^{4t}$$

$$y(1-Ae^{4t}) = 2(1+Ae^{4t}) \Rightarrow y = 2\frac{1+Ae^{4t}}{1-Ae^{4t}}$$

$$y(0) = 0 \Rightarrow 0 = 2\frac{1+A}{1-A} \Rightarrow 0 = \frac{1+A}{1-A} \Rightarrow A = -1.$$

$$y = 2\frac{1-e^{4t}}{1+e^{4t}}$$

19.) $y' = \frac{2t}{1+2y}$ $\quad y(2)=0$

$\int 1+2y\,dy = \int 2t\,dt \Rightarrow y+y^2 = t^2+c$ $\quad y(2)=0 \Rightarrow$

$0+0^2 = 4+c$

$\Rightarrow c=-4$

Quadratic eqn gives $y = \frac{-1+\sqrt{1+4(t^2-4)}}{2}$

23.) $y' = t^2 e^y e^{2t} \Rightarrow \underset{-e^{-y}}{\underset{\parallel}{\int e^{-y}\,dy}} = \int t^2 e^{2t}\,dt$

$\int t^2 e^{2t}\,dt = \frac{1}{2}t^2e^{2t} - \frac{1}{2}\int e^{2t}\,2t\,dt = \frac{1}{2}t^2e^{2t} - \int t e^{2t}\,dt$

$u = t^2 \quad dv = e^{2t}dt$

$du = 2t\,dt \quad v = \frac{1}{2}e^{2t}$

$u = t \quad dv = e^{2t}dt$

$du = dt \quad v = \frac{1}{2}e^{2t}$

$= \frac{1}{2}t^2e^{2t} - \left(\frac{1}{2}te^{2t} - \int \frac{1}{2}e^{2t}\,dt\right)$

$= \frac{1}{2}t^2e^{2t} - \frac{1}{2}te^{2t} + \frac{1}{4}e^{2t} + c$

$= \frac{1}{2}(t^2-t)e^{2t} + \frac{1}{4}e^{2t} + c$

$\Rightarrow e^{-y} = -\left[\frac{1}{2}(t^2-t)e^{2t} + \frac{1}{4}e^{2t} + c\right]$

$y = -\ln\left[-\frac{1}{2}(t^2-t)e^{2t} + \frac{1}{4}e^{2t} + c\right]$

Sect. 1.3

25.) $y' = 1 - y^2$ $\quad 0 = 1 - y^2 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1$

So ⓐ, ⓑ, or ⓒ

$y' = 1$ if $y = 0$. So ⓐ or ⓒ

$y = \pm 2 \Rightarrow y' = -3$ ⓒ

26.) $y' = y^2 - 1$ This is the negative of 25.). ⇒ ⓑ

27.) $y' = y(y-1)(y+1)$ $\quad y = 0, 1, -1$ ⓔ

28.) $y' = (y-1)^2$ $\quad y = 1$

always positive slope ⓕ

29.) $y' = (y+1)(y-1)^2$ $\quad y = \pm 1$ ⓐ, ⓑ, or ⓒ.

ⓑ, ⓒ taken. ⇒ ⓐ

30.) $y' = (y^2+1)(y-1)$

$y = 1 \Rightarrow$ ⓓ