HW #6 SOLUTIONS

$$\left(\begin{array}{l} 3.3: \ 1, 6, 8, 12, 20, 24, 32 \\ 3.4: \ 2, 15, 16, 33 \end{array}\right)$$

SECTION 3.3.

1. If $A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}$ are inverses of each other, then, they should satisfy the equation

$$AB = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Checking for the above property:

$$A \cdot B = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Since $A \cdot B$ turn out to be $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ the pair forms an Inverse pairs.

6. RREF to obtain inverse of $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

* Form the 2×4 matrix $[A | I]$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - 2R_1 \to R_2} \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right] \xrightarrow{-1 \times R_2 \to R_2} \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & 2 & -1 \end{array}\right]$$

$$\xrightarrow{R_1 - 3R_2 \to R_1} \left[\begin{array}{cc|cc} 1 & 0 & -5 & 3 \\ 0 & 1 & 2 & -1 \end{array}\right]$$

Hence, $A^{-1} = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$

8. $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Use RREF to obtain its inverse

$$* \; [A \mid I] = \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\text{Move } R_1 \leftrightarrow R_3.}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right] = [I \mid A^{-1}]$$

$$\therefore \; A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

12. $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix} = A$

Use RREF to obtain its inverse

$$[A \mid I] = \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

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$$\xrightarrow[R_4-R_1\to R_4]{R_3-R_1\to R_3}\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_4+R_2\to R_4]{R_1-R_2\to R_1}\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3+R_4\to R_3]{R_1-R_4\to R_1}\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -2 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2\leftrightarrow R_3}\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -2 & 0 & -1 \\ 0 & 1 & 0 & 0 & -2 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 1 \end{array}\right]$$

20) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}^{-1} \begin{bmatrix} -4 \\ 10 \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 10 \end{bmatrix} = \begin{bmatrix} 50 \\ -18 \end{bmatrix}$

24) $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Use RREF on A.

$\xrightarrow{R_{1/a} \to R_1} \begin{bmatrix} 1 & b/a \\ c & d \end{bmatrix} \xrightarrow{R_2 - cR_1 \to R_2} \begin{bmatrix} 1 & b/a \\ 0 & d - \frac{cb}{a} \end{bmatrix}$

A is investible if $d - \frac{cb}{a} \neq 0 \Rightarrow$

A is not investible if $ad = bc$.

32) $\begin{bmatrix} 1 & 0 & k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Observe that this matrix is upper triangular. Hence, determinant depends only on the diagonal elements. Hence, Determinant $= 1$.

Hence, the above matrix is invertible for any value of k.

SECTION 3.4.

2). Use row 2 to perform the Cofactor expansion

Hence $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & -3 \end{vmatrix} = +1 \begin{vmatrix} 1 & 3 \\ 1 & -3 \end{vmatrix} = -6.$

15). Use a 4×4 matrix to verify this

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$

To find $|A|$, do a cofactor expansion along column 1.

$\therefore |A| = a_{11} \underbrace{\begin{vmatrix} a_{22} & a_{23} & a_{24} \\ 0 & a_{33} & a_{34} \\ 0 & 0 & a_{44} \end{vmatrix}}_{= |B|}$

To find $|B|$, again do a cofactor expansion along column 1.

$|B| = a_{22} \begin{vmatrix} a_{33} & a_{34} \\ 0 & a_{44} \end{vmatrix} = a_{22} \times a_{33} \times a_{44}$

$\therefore |A| = (a_{11})(a_{22})(a_{33})(a_{44})$

Hence proved.

16). $|A| = \begin{vmatrix} -3 & 4 & 0 \\ 0 & 7 & 6 \\ 0 & 0 & 5 \end{vmatrix}$ Observe that A is an upper triangular matrix.

From (15), $|A|$ = product of diagonal elements of upper triangular matrix.

Hence, $|A| = (-3)(7)(5) = -105$

33) Choose $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$|A| = 1$ $|B| = 1$

$A+B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $|A+B| = 0$

Hence, $|A+B| \neq |A| + |B|$.