Example $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ Solve $\dot{\mathbf{x}} = A\mathbf{x}$!

Sol^n $p(\lambda) = \det(A - \lambda I) = \det\begin{bmatrix} 1-\lambda & 1 \\ -1 & 1-\lambda \end{bmatrix} = (1-\lambda)^2 + 1 \Rightarrow \lambda = 1 \pm i$

$\lambda_1 = 1+i \qquad \lambda_2 = 1-i$

Find $\mathbf{v}_1$!

$$\left[\begin{array}{cc|c} 1-(1+i) & 1 & 0 \\ -1 & 1-(1+i) & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} -i & 1 & 0 \\ -1 & -i & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} -i & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & i & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\mathbf{v}_1 = \begin{bmatrix} -i \\ 1 \end{bmatrix}$ Verify! $A\mathbf{v}_1 = \begin{bmatrix} -i+1 \\ i+1 \end{bmatrix} = \begin{bmatrix} -i(1+i) \\ (1+i) \end{bmatrix} = (1+i)\begin{bmatrix} -i \\ 1 \end{bmatrix}$

$\mathbf{p} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \mathbf{q} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$\mathbf{y}_1 = e^t\left(\cos(t)\begin{bmatrix} 0 \\ 1 \end{bmatrix} - \sin(t)\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} e^t\sin(t) \\ e^t\cos(t) \end{bmatrix}$

$\mathbf{y}_2 = e^t\left(\sin(t)\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \cos(t)\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -e^t\cos(t) \\ e^t\sin(t) \end{bmatrix}$

General sol^n is $\mathbf{y}(t) = \begin{bmatrix} c_1e^t\sin(t) - c_2e^t\cos(t) \\ c_1e^t\cos(t) + c_2e^t\sin(t) \end{bmatrix}$