APPM 2360: Section exam 1
7.00pm – 8.30pm, February 11, 2009.

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) recitation section (4) your instructor's name, and (5) a grading table. Text books, class notes, and calculators are NOT permitted. A one-page crib sheet is allowed.

Problem 1: (28 points) For this problem, give the answer only. No motivation is required. Each correct answer earns 4 points (no partial credits will be given).

(a) Give the *general* solution to the equation $y' = -t\,y^2$.

(b) Give a solution to the equation $y' = -t\,y^2$ that satisfies $y(0) = 7$.

(c) Identify the equilibrium points to the equation $y' = y^3 - y$.
(You do not need to determine whether they are stable.)

(d) Consider the initial value problem

$$\begin{cases} y' &= (1 + t\,y)\,y \\ y(0) &= 7. \end{cases}$$

Compute an approximation to $y(1/2)$ by performing *one step* of the forwards Euler method (in other words, use the step size $h = 1/2$).

(e) Which of the following equations are both linear and homogeneous:
(1) $y'' + t^2\,y = 0$
(2) $y'' + t\,y^2 = 0$
(3) $y' + 2\,y + 1 = 0$
(4) $y' + 2\,y + t^2 = 0$.

(f) Determine all values of a for which Picard's theorem guarantees that the following initial value problem has at most one solution:

$$\begin{cases} y' &= t\,y^a \\ y(1) &= 0. \end{cases}$$

(g) The equation

$$y' + 2\,y = 8t^3 + 2 \tag{1}$$

has the particular solution

$$y_{\text{p}} = 4t^3 - 6t^2 + 6t - 2$$

Find the solution y of equation (1) that satisfies $y(0) = 2$.

For question 2 — 4, motivate your answers. A correct answer with no work may receive no credit, while an incorrect answer with some correct work may result in partial credit.

Problem 2: (18 points) Consider the equation $y' + t\,y = t$.

(a) (9 points) Construct the general solution.

(b) (9 points) Construct the solution that satisfies $y(1) = 2$.

Problem 3: (18 points) Consider the equation

$$y' = y - \frac{y}{t} + \frac{1}{t}. \tag{2}$$

(a) (9 points) Set $z(t) = t\,y(t)$. Construct a differential equation for z that is equivalent to equation (2).

(b) (9 points) Construct the general solution to (2).

Problem 4: (18 points)

(a) (6 points) A radioactive material, Dirtyum, is known to decay at a rate proportional to the amount present. It takes 2 years for 10 kg to decay down to 1 kg. Find $A(t)$, the amount of Dirtyum as a function of time, if there are 100 tons of Dirtyum at time $t = 0$.

(b) (6 points) Initially ($t = 0$), 100 tons of Dirtyum are deposited in an underground storage facility. In addition to this initial amount, more Dirtyum is dumped continuously in the facility at a rate totaling 5 tons per year. Write down a differential equation for $A(t)$ and an initial equation describing this situation.

(c) (6 points) Solve the differential equation in part (b) to find $A(t)$ and find what is the amount of Dirtyum in the facility as $t \to \infty$.

Note: Your answers may contain unevaluated formulas (for instance, "as $t \to \infty$, the amount of dirtyum will approach $3\cos(\log(4))$ tons").

Problem 5: (18 points) For this problem, simply state the answer, no motivation is necessary. Consider the following first order ordinary differential equations:

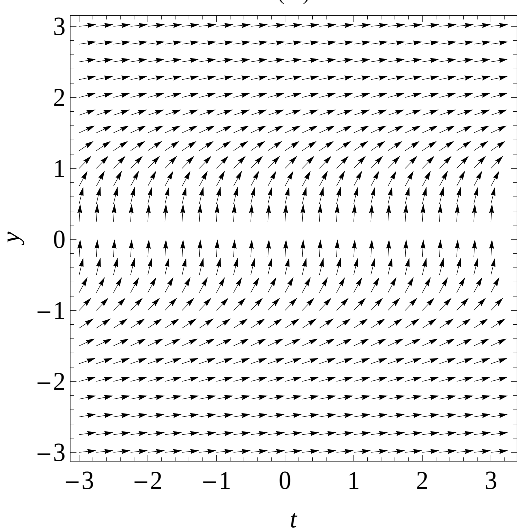
(1) $y' = y + t^2,$ (2) $y' = y^2,$ (3) $y' = \sin(y),$

(4) $y' = y^2 - 4,$ (5) $y' = y^2 + 2\,y,$ (6) $y' = 1/y^2.$

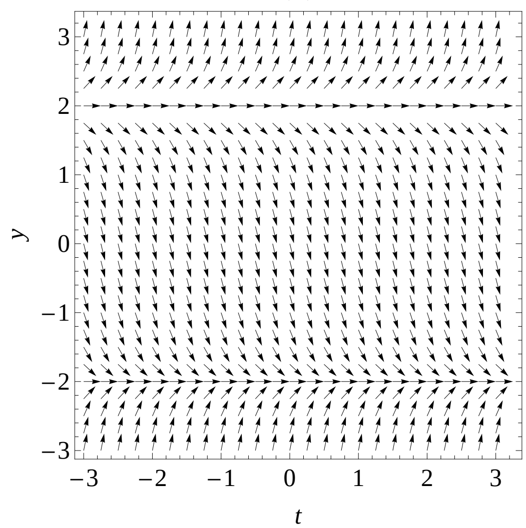
(a) (9 points) Match direction fields (A)—(D) to the above differential equations. (Note that there are more equations than direction fields, so two equations have no corresponding direction fields.)

(b) (9 points) Identify the equilibria shown in **EACH GRAPH** and give their stability.

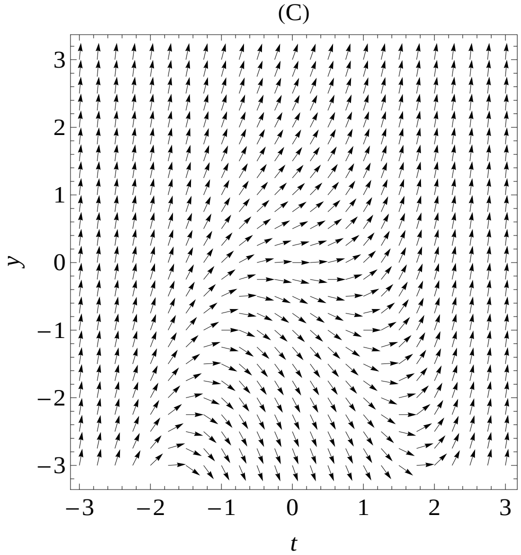
(A)



(B)



(C)



(D)