Exam #2 **APPM 2360** **Spring 2008**

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, deficient (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and box in your final answer. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are NOT permitted. A one-page crib sheet is allowed.

1. (*20 points, 4 each*) State whether the following statements are *always* "TRUE" or "FALSE" (meaning not *always* true). You MUST write the full word TRUE or FALSE. T/F or YES/NO will NOT be given any credit. You do not have to justify your answer for this question.

(a) For all $n \times n$ matrices, $(AB)^T = A^T B^T$, where A^T and B^T are the transposes of A and B.

(b) If A is an $n \times n$ triangular matrix with a zero element on the diagonal, then $|A| = 0$.

(c) If the square system $A\mathbf{x} = \mathbf{0}$ has a unique solution, then $A^T\mathbf{x} = \mathbf{0}$ also has a unique solution.

(d) If $\mathbb{W}$ is a vector subspace of the vector space $\mathbb{V}$, then $\mathbb{W}$ contains the zero vector $\mathbf{0}$.

(e) If $|A| \neq 0$ for an $n \times n$ matrix A, then the columns of A form a basis for $\mathbb{R}^n$.

2. (*24 points, 8 each*) Consider the matrix $F(x) = \begin{bmatrix} 1 & 0 \\ x & 1 \end{bmatrix}$ with $x \in \mathbb{R}$.

(a) Show that $F(x) \cdot F(y) = F(x + y)$ and find F^{-1}.

(b) Show that $\dfrac{dF}{dx} + \dfrac{d(F^{-1})}{dx} = \mathbb{O}$, where $\mathbb{O}$ is the 2×2 zero matrix.

(c) Set $B = \dfrac{dF}{dx}$ and evaluate the exponential of the matrix B using the series $e^B = I + \displaystyle\sum_{n=1}^{\infty} \frac{B^n}{n!}$, where I is the 2×2 identity matrix.
Hint: Calculate B^2.

3. (*21 points, 7 each*) The matrices

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 & -1 \\ 6 & 3 & -1 & 0 & 0 \\ -2 & -1 & -1 & 1 & 5 \\ 4 & 2 & 0 & -2 & -4 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 2 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

are row equivalent.

(a) Find the rank of A.

(b) Give a basis for the solution space of the system $A\mathbf{x} = \mathbf{0}$.

(c) Does the equation $A\mathbf{x} = \mathbf{b}$ have a solution for every $\mathbf{b} \in \mathbb{R}^4$?

4. (*21 points, 7 each*) Consider the equation $z^2 - \lambda z + 1 = 0$ and the system $\begin{cases} \lambda x + 4y = 0 \\ x + \lambda y = 0 \end{cases}$.

(a) Find the values of λ for which the system has a unique solution and calculate that solution.

(b) Show that if the equation has a double root then the system has infinitely many solutions.

(c) Fix a value of λ for which the system has infinitely many solutions. Let z_0 denote the corresponding double root of the equation. Prove that if (x_0, y_0) is a solution of the system, then $x_0^2 + (1 - 5\,z_0^2)\,y_0^2 = 0$.

More on the back; turn over the page.

5. (*24 points, 8 each*) Consider the set $\mathbb{W}$ of all 3×3 diagonal matrices of the form $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ where a, b, and c are real constants.

(a) Show that $\mathbb{W}$ is a subspace of the vector space $\mathbb{V}$ of all 3×3 matrices.

(b) Show that the matrices

$$A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

form a basis for $\mathbb{W}$ and find the dimension of $\mathbb{W}$.

(c) Consider the matrix $A_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Do A_1, A_2 and A_4 form a basis for $\mathbb{W}$?

Extra credit: (*5 points*) If the matrix $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ is *not* invertible calculate A^n, $n > 1$.

Good Luck!!!