APPM 5440, Fall 2005.

Notes on Chapter 4 – Point set topology

Core topics:
These topics may show up on an exam. You are expected to know these definitions and results.

- Definition of a topological space. Definition of open and closed sets.
- Topologies generated by metrics. The concept of metrizability.
- Topologies induced on subsets.
- The Hausdorff property. Every metric space is Hausdorff.
- Definition of convergence of sequences.
- Definitions of continuous and open maps.
- Definition of homeomorphisms.
- Definition of compact sets. The continuous image of a compact set is compact.
- The definition of open bases, and neighborhood bases.
- The definition of second countable spaces. The relationship between second countable spaces and separable spaces.
- The concept of weaker and stronger topologies. Strong convergence implies weak convergence. Weak continuity implies strong continuity (*i.e.*, if $\mathcal{T}_1 \subseteq \mathcal{T}_2$, and if $f\colon (X, \mathcal{T}_1) \to (Y, \mathcal{S})$ is continuous, then $f\colon (X, \mathcal{T}_2) \to (Y, \mathcal{S})$ is continuous).

Other topics:
You should be familiar with these topics since they relate to material we will cover later in class. There may be exam questions related to these topics, but you are not expected to know the definitions or the results by heart.

- The closure of a set. The sequential closure of a set. The boundary and the interior of a set (see Exercise 4.2).
- Compact sets are closed in Hausdorff spaces.
- Sub-bases. The topology induced by a sub-base. The topology induced by a function or a set of functions.
- First countable spaces.
- Relationship between a base and a neighborhood base.
- For metric spaces, two topologies are equivalent if and only if they generate identical convergent sequences.