

Homework set 1 — APPM5440, Fall 2006

From the textbook: 1.3, 1.4, 1.5.

Problem 1: Consider the set \mathbb{R}^n equipped with the norm

$$\|x\|_p = \left(\sum_{j=1}^n |x_j|^p \right)^{1/p}.$$

- (a) Prove that $\|\cdot\|_p$ is a norm for $p = 1$.
- (b) Prove that $\|\cdot\|_p$ is a norm for $p = 2$.
- (c) Prove that $\lim_{p \rightarrow \infty} \|x\|_p = \max_{1 \leq j \leq n} |x_j|$.

Problem 2: Set $I = [0, 1]$ and consider the set X consisting of all continuous functions on I . Define an addition and a scalar multiplication operator that make X a normed linear space.

(a) Which of the following candidates define a norm on X :

- $\|f\|_a = \sup_{0 \leq x \leq 1} |f(x)|$
- $\|f\|_b = \sup_{0 \leq x \leq 1/2} |f(x)|$
- $\|f\|_c = \sup_{0 \leq x \leq 1} |f(x)|^2$
- $\|f\|_d = 2 \sup_{0 \leq x \leq 1} |f(x)|$
- $\|f\|_e = \sup_{0 \leq x \leq 1} (1 + \cos x)|f(x)|$
- $\|f\|_f = |f(0)| + \sup_{0 \leq x \leq 1} |f(x)|$
- $\|f\|_g = |f(0)|$

(b) Prove that

$$\|f\| = \int_0^1 |f(x)| dx$$

is a norm on X .

(c) Prove that with respect to the norm given in (b), the space X is not complete.