

Homework set 4 — APPM5440

From the textbook: 1.27, 2.2, 2.3.

Problem 1: We define a subset Ω of \mathbb{R} via

$$\Omega = \{0\} \cup \left(\bigcup_{n=1}^{\infty} \left[\frac{1}{n+1/2}, \frac{1}{n} \right] \right).$$

Prove that Ω is compact.

Problem 2: Show that on any set X , uniform convergence implies pointwise convergence.

Problem 3: Let X be a finite set. Show that pointwise convergence on X implies uniform convergence.

Problem 4: Let X be an infinite set. Construct a sequence of functions $f_n : X \rightarrow \mathbb{R}$ that converges pointwise, but does not converge uniformly.