Homework set 9 — APPM5440

From the textbook: 4.1, 4.2, 4.3.

Problem: The Hausdorff property is only one of many so called "separability" conditions on topological spaces. As an example, we say that a topological space X is T_j, for $j = 0, 1, 2, 3, 4$ if:

- T_0**:** Given $x, y \in X$, there either exists an open set containing x but not y, or vice versa.
- T_1**:** Given $x, y \in X$, there exists an open set that contains x but not y.
- T_2**:** Given $x, y \in X$, there exist disjoint open sets G, H such that $x \in G$, $y \in H$. (Note that T_2 is the same as Hausdorff.)
- T_3**:** X is T_1, and: Given any closed set A, and any point $x \in A^c$, there exist disjoint open sets G, H such that $x \in G$, $A \subseteq H$.
- T_4**:** Given any two closed disjoint sets A and B, there exists disjoint open set G, and H such that $A \subseteq G$, $B \subseteq H$.

Prove that if $i < j$, then any T_j space is T_i. Prove that the co-finite topology is T_1 but not T_2. Prove that a topological space is T_1 if and only if the set $\{x\}$ is closed for every $x \in X$.