APPM 5440, Fall 2012.

Notes on Chapter 4 – Point set topology

Core topics:

These topics may show up on an exam. You are expected to know these definitions and results well.

- Definition of a topological space. Definition of open and closed sets.
- Topologies generated by metrics. The concept of metrizability.
- Topologies induced on subsets.
- The Hausdorff property. Every metric space is Hausdorff. Compact sets are closed in Hausdorff spaces.
- Definition of convergence of sequences.
- Definitions of continuous and open maps.
- Definition of homeomorphisms.
- Definition of compact sets. The continuous image of a compact set is compact.
- For metric spaces, two topologies are equivalent if and only if they generate the same convergent sequences.
- The concept of weaker and stronger topologies. Strong convergence implies weak convergence. Weak continuity implies strong continuity (*i.e.*, if $\mathcal{T}_1 \subseteq \mathcal{T}_2$, and if $f\colon (X, \mathcal{T}_1) \to (Y, \mathcal{S})$ is continuous, then $f\colon (X, \mathcal{T}_2) \to (Y, \mathcal{S})$ is continuous).

Other topics:

You should be familiar with these topics since they relate to material we will cover later in class. There may be exam questions related to these topics, but they will not ask you to carry out extensive arguments beyond the definitions. If you find yourself short on time for preparing, make sure that you understand the core topics before working on these.

- The closure of a set. The sequential closure of a set. The boundary and the interior of a set (see Exercise 4.2).
- The definition of open bases, and neighborhood bases.
- Sub-bases. The topology induced by a sub-base. The topology induced by a function or a set of functions.
- The definition of second countable spaces. The relationship between second countable spaces and separable spaces.
- First countable spaces.
- Relationship between a base and a neighborhood base.