Homework set 4 — APPM5440 Fall 2016

From the textbook: 2.2, 2.3, 2.4, 2.5.

Problem 1: Show that on any set X, uniform convergence implies pointwise convergence.

Problem 2: Let X be a finite set. Show that pointwise convergence on X implies uniform convergence.

Problem 3: Let X be an infinite set. Construct a sequence of functions $f_n : X \to \mathbb{R}$ that converges pointwise, but does not converge uniformly.

Problem 4: Let $X = [0, \infty)$. Construct a sequence of functions $f_n : X \to \mathbb{R}$ that converges uniformly (and hence pointwise), but that does not converge in $L^2(X)$.

Problem 5: Let $X = [0, 1]$. Construct a sequence of functions $f_n : X \to \mathbb{R}$ that converges in $L^2(X)$ but such that the sequence of numbers $(f_n(x))_{n=1}^{\infty}$ does not converge for *any* $x \in X$.