Homework set 8 — APPM5440 — Fall 2016

From the textbook: 3.6, 3.7.

Problem 1: Let X be a set with infinitely many members. We define a collection $\mathcal{T}$ of subsets of X by saying that a set $\Omega \in \mathcal{T}$ if either $\Omega^{\rm c} = X\backslash\Omega$ is finite, or if Ω is the empty set. Verify that $\mathcal{T}$ is a topology on X. This topology is called the "co-finite" topology on X. Describe the closed sets.

Problem 2: Let X denote a finite set, and let $\mathcal{T}$ be a metrizable topology on X. Prove that $\mathcal{T}$ is the discrete topology on X.

Problem 3: Consider the set $X = \{a,\, b,\, c\}$, and the collection of subsets $\mathcal{T} = \{\emptyset,\, \{a\},\, \{a,b\},\, \{a,c\},\, \{a,b,c\}\}$. Is $\mathcal{T}$ a topology? Is $\mathcal{T}$ a metrizable topology?

Problem 4: Consider the integral equation

$$(*) \qquad u(x) = \pi^2 \sin(x) + \frac{3}{2}\int_0^{\cos(x)} |x-y|\, u(y)\, dy.$$

Prove that (*) has a unique solution in $C([0,1])$.