Homework set 10 — APPM5440 — Fall 2016

5.1, 5.3, 5.4, 5.5 (feel free to assume that k is non-negative), 5.7, 5.8.

Problem 1: Set $X = \mathbb{R}^n$, $Y = \mathbb{R}^m$, and let $A \in \mathcal{B}(X, Y)$. Let

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

denote the representation of A in the standard basis.

(a) Equip X and Y with the ℓ^1 norms. Compute $||A||$.

(b) Equip X and Y with the ℓ^2 norms. Compute $||A||$.

(c) Equip X and Y with the ℓ^∞ norms. Compute $||A||$.

Problem 2: Suppose that X is a NLS and that Y is a Banach space. Let Ω be a dense subspace of X, and let

$$T : \Omega \to Y$$

be a linear function such that

$$M = \sup_{x \in \Omega,\ x \neq 0} \frac{||Tx||}{||x||} < \infty.$$

In other words, M is the norm of T, viewed as a map from Ω. Prove that there exists a unique linear map $\bar{T} : X \to Y$ such that $\bar{T}x = Tx$ for every $x \in \Omega$. Prove that $||\bar{T}|| = M$.