**Hints for homework set 11 — APPM5440 — Fall 2016**

**Problems 5.11 and 5.12:** Should be fairly straight-forward.

**Problem 5.13:** (a) $\|A\| = \max_n |\lambda_n|$, $\|N\| = 1$.

(b) $$\exp(\Lambda t) = \begin{bmatrix} e^{\lambda_1 t} & 0 & 0 & \cdots \\ 0 & e^{\lambda_2 t} & 0 & \cdots \\ 0 & 0 & e^{\lambda_2 t} & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}.$$

For $N$, observe that $N^n = 0$ so the defining sum is finite:

$$\exp(Nt) = I + tN + \frac{1}{2}t^2\, n^2 + \cdots \frac{1}{(n-1)!}\, t^{n-1}\, N^{n-1}.$$

Work out for yourself what $N^j$ looks like!

**Problem 5.15(a):** You can look this proof up in many places on the web.

**Problem 5.17:** Define $A_n = \sum_{j=0}^{n} K^j$. Then prove that in norm, you have both that $(I-K)A_n \to 0$ and that $A_n(I-K) \to 0$.

**Problem 1:** Hmm, this is just 5.15(a). My mistake ...

**Problem 2:** Prove that $\|T_n\| = 1/\sqrt{n}$. Once you do that, it follows immediately that $T_n \to 0$ in both norm and strongly.