Description of Matlab's Fast Fourier Transform Interface

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The relationship between the time dependent sound data y_j and the frequency dependent c_k produced by Matlab's fft function is

$$\begin{aligned} y_j &= \frac{1}{N}\sum_{k=1}^{N} c_k e^{2\pi i \frac{j-1}{N}(k-1)} && (1)\\ &= \frac{1}{N}\sum_{k=1}^{N} c_k \cos\left(k-1\right)\tau_j + i c_k \sin\left(k-1\right)\tau_j, \quad \tau_j = 2\pi\frac{j-1}{N}, \quad j = 1\ldots N. && (2)\end{aligned}$$

In a domain with N points $\cos k\tau_j$ looks just like $\cos\left(N-k+2\right)\tau_j$ and $\sin k\tau_j$ looks just like $-\sin\left(N-k+2\right)\tau_j$ on $j = 1\ldots N$, as shown in Figure 1. This phenomena, called aliasing, must be accounted for when analyzing the sound file. Rearranging the sum and setting the average $c_1 = 0$,

$$y_j = \frac{1}{N}\sum_{k=2}^{N/2}\left(c_k + c_{N-k+2}\right)\cos\left(k-1\right)\tau_j + \left(c_k - c_{N-k+2}\right) i \sin\left(k-1\right)\tau_j. \qquad (3)$$

Note that if $\{y_j\}_{j=1}^{N}$ is real then $\mathrm{Im}\left(c_k + c_{N-k+2}\right) = 0$ and $\mathrm{Re}\left(c_k - c_{N-k+2}\right) = 0$, implying that $c_k = \overline{c_{N-k+2}}, \quad k = 2\ldots N/2$. The power in the k^{th} frequency of the power spectrum is then

$$\begin{aligned} p_k &= \sqrt{a_k^2 + b_k^2} = \sqrt{\left(c_k + c_{N-k+2}\right)^2 + \left(i\left(c_k - c_{N-k+2}\right)\right)^2} \\ &= \sqrt{4 c_k c_{N-k+2}} = 2\left|c_k\right|, \qquad k = 2\ldots N/2. \end{aligned} \qquad (4)$$

Thus the Matlab code `c = fft(y(1:N))/N` and `p = 2*abs(c(2:N/2))` converts the sound data vector `y` into the vector `p`, the amplitude of each harmonic.

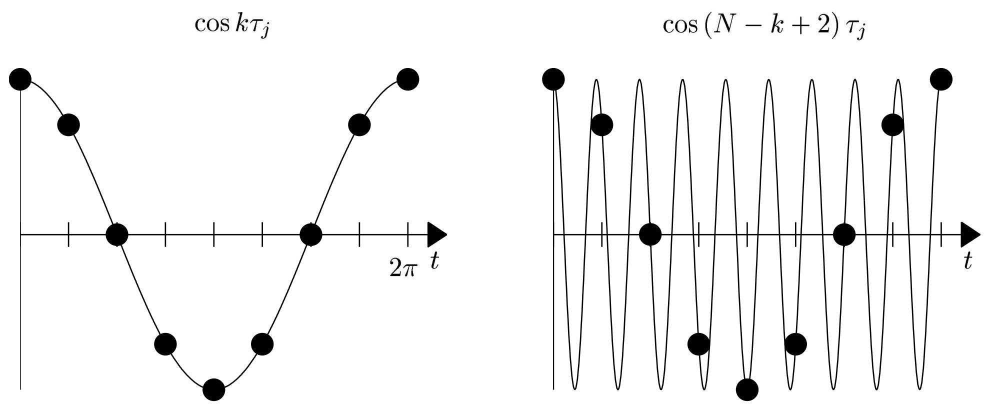

Figure 1: The effect of aliasing. Here $N = 8$, $k = 1$ and $\tau_j = 2\pi\frac{j-1}{N}$, $\; j = 1 \ldots N$.