# ERRATA AND ADDITIONS: SECOND EDITION July 2010

# COMPLEX VARIABLES, INTRODUCTION AND APPLICATIONS

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Corrections and small additions

**p. 4** line 8 from top: after "..by arg z." add "We take the standard convention, counterclockwise is the positive direction."

**p.7** Problem 2c, replace:  $\cos(z) = (e^{iz} + e^{-iz})/(2)$  by  $\cos z = (e^{iz} + e^{-iz})/2$ 

p.8 Problem 4. Spelling: change: "Estabilish" to "Establish"

p.33 16 lines from top: eliminate "exist and"; now reads: "...must be differentiable.."

**p.34**: line 4 from top: eliminate "after noting Eq. (2.1.2) and manipulating." and replace " $|\Delta z|$  approaching zero" by " $|\Delta x|$ ,  $|\Delta y|$  approaching zero".

**p.39** In Example 2.1.6, 1st line after equations replace " $\psi(x)$ " by " $\psi(y)$ "

**p.47** line 8 from bottom: just before "The semiaxis.." add "In fact , for  $0 < \theta_p < 2\pi, r \neq 0$ , the function is analytic (from the Cauchy-Riemann conditions)."

**p.48** 4 lines from top, replace "On the other hand, if we took.." by "On the other hand we reiterate, if we took..."

**p.66** line 11 from top, replace: "Riemann surface with a cut along the positive x-axis." by "Riemann surface on which each sheet connects smoothly."

**p.71** lines 8,9 from bottom, replace "..there is a set of points...of the interval." to: "..there is a point (x(t), y(t)) that yields the image point z(t).

# p.76

line 4 from bottom, after  $dz=rie^{i\theta}d\theta$  add: " and since the curve is simple we can take  $0<\theta<2\pi$ 

(on same page) line 2 from bottom, replace "any closed curve" by "any simple closed curve"

**p.84** 2 lines from bottom, replace "over z" by " of f(z) over the simple contour C

**p.88** line 10 from top, replace "a small, but finite circle of radius r" by "a small circle of radius r > 0

**p.98** line 8 from top: after harmonic functions" add: u = Ref or v = Imf, f = u + iv,

**p.99** In Theorem 2.6.7: change: "... bounded by a simple closed contour C, then at any interior point z" to "... bounded by a simple closed contour C, and if f is continuous on C, then at any interior point z"

# p.111

line 6 in proof of Theorem 3.1.1, after "n > N," add: "N independent of z"

(on same page) line 8 in proof of Theorem 3.1.1, before "Continuity" add: "Now take an n > N."

(on same page) line 10 in proof of Theorem 3.1.1, omit "n > N,"; line should now read: "Thus for  $|z - z_0| < \delta$ "

# p.113

Line 3 change: "Theboundedness..." to "The boundedness..."

(on same page) In the two equations following line 3, change: " $|b_1(z)| < B$  hence  $|b_n(z)| < BM^{n-1}$ " to " $|b_1(z)| \le B$  hence  $|b_n(z)| \le BM^{n-1}$ "

(on same page) line 5 above Problems, after Theorem 3.1.2 add: "(proven in section 3.4)"

**p.114** Problem 5b, replace " $R < |Re \ z| \le 1$ " by " $R < Re \ z \le 1$ "

**p.131** line 10 from top: replace equation:  $f(z) = \sum_{n=-\infty}^{0} \frac{C_n}{(z-z_0)^n}$  by  $f(z) = \sum_{n=0}^{\infty} \frac{C_n}{(z-z_0)^n}$ 

**p.135** Near top of page; in the formula for  $C_n$ , change  $\frac{1}{2^{n+1}}$  to  $\frac{-1}{2^{n+1}}$ 

**p.137** Problem 7; in the formula for  $R_n(z)$  change  $(-)^{n+1}$  to  $(-1)^{n+1}$ 

#### p.139

line 1 of Theorem 3.4.3 replace "... $\leq M_j$  in some region R with  $M_j$  a sequence of constants." by ""... $\leq M_j$ , with each  $M_j$ , j = 1, 2... constant, in some region R." (on same page) last line of proof replace "Theorem 3.1.2 follows." by "Theorem 3.1.3 follows."

### p.141

line 6 from top replace " $|\zeta - z| > \nu$ " by " $|\zeta - z| \ge \nu$ "

(on same page) line 11 from top after "uniformly" add tof'(z) and after "and" add "then employing"

(on same page) line 14 from top eliminate "all"

**p.142** line 5 from top before "has no" add: " $x \neq m\pi, m$  integer,"

**p.145** Example 3.5.2, first line. Replace: "Describe the singularities of the function" by "Describe the singularity of the function at z = 0"

**p.146** Example 3.5.3 The line after the formula, change: "Here the function f(z) has simple poles with strength 1..." to "Here we will show the function f(z) has simple poles with strength -1..."

#### **p.148**

3 lines above Eq. (3.5.5) after "...  $f(z) \to 0$  as  $r \to 0$ ." add: "Also for  $\theta = \pm \pi/2$ , |f(z)| = 1."

(on same page) 2 lines above Eq. (3.5.5) change "... namely,  $r = (1/R) \cos \theta$  (i.e the points..." to "... namely,  $r = (1/R) \cos \theta$ ,  $R \neq 0$ , (i.e., the points..."

(on same page) last two lines, change: "Thus |f(z)| may take on any positive value other than zero by the appropriate choice of R" to "Thus |f(z)| may take on any positive value in the neighborhood of z = 0".

**p. 162** First line; change: "If we assume that  $|a_k(z)| \leq M_k...$ " to "If we assume that  $|a_k(z)| < M_k...$ "

#### p.181

In Theorem 3.7.3: change: "... simply connected domain D, then the linear..." to "... simply connected domain D containing  $z_0$ , then the linear..."

#### p.185

line after Eq. (3.7.41), before: " $(z = 0 \text{ can be translated to } z = z_0 \text{ if we wish})$  insert: " $z \neq 0, \omega_{m,n}$ "

(on same page) line after Eq. (3.7.43), before "The function ..." insert: "Alternatively, by

taking the derivative of Eq.(3.7.42) w satisfies " $w'' = 6w^2 - \frac{g_2}{2}$ ".

**p.186** line immediately after Eq. (3.7.45) insert (no new paragraph): "Also note that  $w_1$  satisfies the second order ODE  $w_1'' = 2k^2w_1^3 - (1+k^2)w_1$ ."

**p.198** 2nd line above Example 3.8.2 change "... time T with ..." to "... distance with  $\dots$ "

**p.206** 3rd line from bottom change "... lying in D." by "...lying in D and enclosing  $z_0$ ."

**p.240** after  $z = z_0 + \epsilon e^{i\theta}$  insert: " and taking  $\theta$  from 0 to  $\phi$ ,

**p.257** Problem 14, 3rd line, change: "... where  $C_R$  is the ...." to "... where  $C_R$  is the outside part of the ...."

**p.258** Problem 14, part (c): left hand side add dx in the integrand; change the sign of the right hand side: from "=  $\pi b_{n+2}$ " to "=  $-\pi b_{n+2}$ "

**p.266** problem 6. Change the last two lines from: "Consider the two functions  $-f_0$  and  $f(z) - f_0$ , and use ... to deduce that  $f(z) = f_0$ ." to: "Consider the two functions  $-f_0$  and f(z). Then Rouché's Theorem implies that the functions  $-f_0$ ,  $f(z) - f_0$  have the same number of zeroes.

#### p.268

line 10-11 from top, omit: "(sometimes referred to as bounded mean oscillations (BMO))"; (on same page) line 12 from top, omit "(i.e. in BMO)"

**p.270** In the right hand side of eq. (4.5.10) replace  $\delta(x - x_0)$  by  $\Delta(x - x_0; \epsilon)$ 

# p.272

In eq. (4.5.17) 2nd line replace  $e^{ikx'}g(x')$  by  $e^{-ikx'}g(x')$ (on same page) 2 lines after eq. (4.5.18) replace  $f(x) = \delta(x - x')$  by  $f(x) = \delta(x)$ 

**p.329** line 8 from top before "(See also...)." add "Note, in the above Eq. when  $x^2 + y^2 \rightarrow 1$ , for y > 0, y < 0,  $\tan^{-1}[\cdot] \rightarrow 0, \pi$  respectively."

#### p.341

line 10 from bottom replace "one value for  $g(z) = w - w_0 = a$  corresponding to every z inside" to "one value z for which  $g(z) = w - w_0 = a$  inside" (on same page) replace 'Rouche" by "Rouché" everywhere

#### p.342

line 14 from bottom replace " $< \epsilon_1$  and therefore F(w) is continuous." by " $< \epsilon_1$ . Now let  $\delta_1$  be small enough so that  $|w - w_1| < \delta_1$  is in P and  $|F(w) - F(w_1)| < \epsilon_1$ . Since  $\epsilon_1$  is arbitrary and there is a corresponding  $\delta_1 > 0$ , therefore F(w) is continuous."

(on same page) line 6 from bottom after "only one solution" add " counting multiplicity"

**p.343** line 4 in proof replace "to an arbitrary point  $z_0 \in D$  and is not a point ..." to "to a point  $z_0 \in D$  which is not a point ..."

**p.401** line 6 from bottom in the equation for g(z) + 1 replace  $(z \cdot z_2)^3$  by  $(z - z_2)^3$ 

**p.426** In Example 6.2.4 replace  $e^{-\frac{t}{k}}$  by  $e^{-\frac{t}{\epsilon}}$ 

**p.463** In Example 6.4.5 line above first set off equation: replace  $y \to \pm i\pi$  by  $y \to \pm \pi$ 

**p.471** In two places on the page (middle of the page and after formulae for  $I_1(k)$ ) spelling correction: change "principle value" to "principal value"

**p.515** in Eq. (7.1.8) replace z - t by t - z in the denominator of the integral

**p.520** line 4 from bottom replace  $t \to \pm \infty$  by  $\tau \to \pm \infty$ 

p.522 line below Eq. (7.2.16) replace "part 1" by "part 1" -i.e. eliminate bold on 1.

**p.529** line 2 from bottom replace "g(t) is  $\kappa$ " by "g(t) is  $\kappa \neq 0$ "

**p.528** line 1 of Example 7.3.1 replace "inside a closed contour" by "inside a simple closed contour"

**p.563** First equation in 2nd paragraph for  $\Phi(k)$ . Inside integral (add a left parens.): change  $\frac{f(l)}{X^+(l)l-k}$  to  $\frac{f(l)}{X^+(l)(l-k)}$ 

**p.606** line 5 from bottom replace  $\mathbf{g} \to \hat{\mathbf{f}}$  and  $\to \hat{\mathbf{h}}$  by  $g \to \hat{g}$  and  $h \to \hat{h}$  –note: no boldface necessary.

**p. 619** in the equation 3 lines below equation (7.7.26b) replace

$$\Psi_{\mathbf{1}} \sim \left(\frac{1}{k}\right)^{\theta} \quad \text{by} \quad \Psi_{\mathbf{1}} \sim \left(\frac{1}{k}\right)^{\theta} \begin{pmatrix} 1\\ 0 \end{pmatrix}$$

**p.630** In the solution 3b, move parentheses: from left of summation to just inside summation; i.e. from  $(\sum_{n=0}^{\infty} \dots \text{ to } \sum_{n=0}^{\infty} (\dots$ 

 $\mathbf{p.640}$  index: omit reference to 'BMO (bounded mean oscillations), 268"