Solitons in mode–locked lasers

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- Ti:Sapphire lasers

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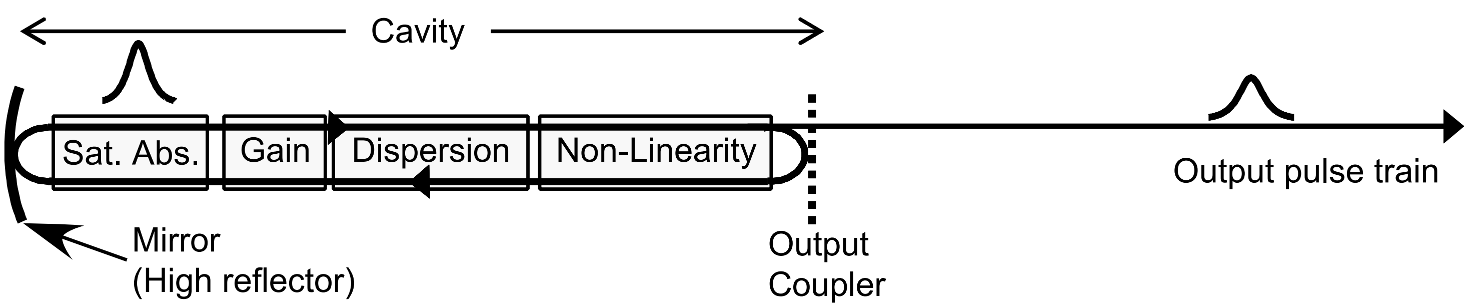
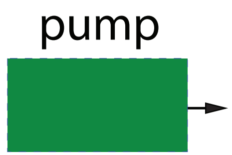
- Damping is provided by the laser cavity, which is a set of mirrors that cause light to reflect on itself.
- Mode–locking: A frequency domain description of how ultra short pulses are generated by the laser.
- Schematic diagram of the elements present in a mode–locked laser.



The mechanism of the laser.

- Pump: The pump emits green light from either an Ar^{+} laser or diode–pumped solid state laser.



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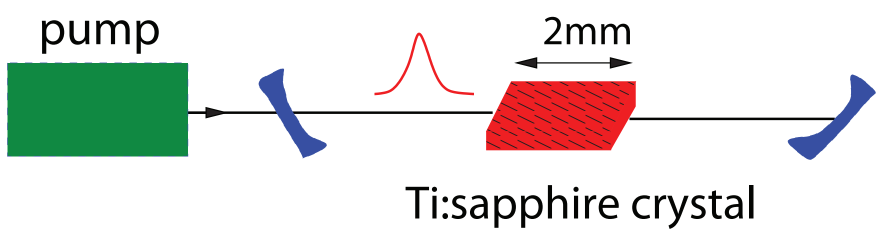

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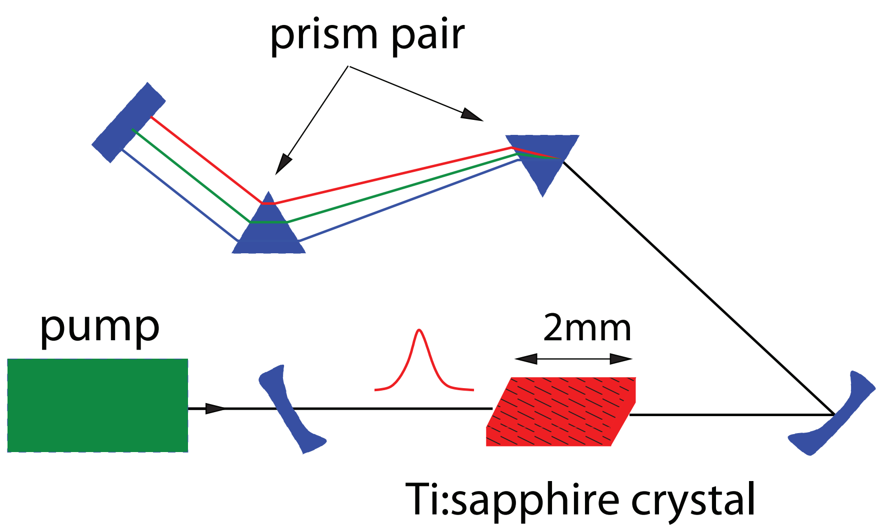

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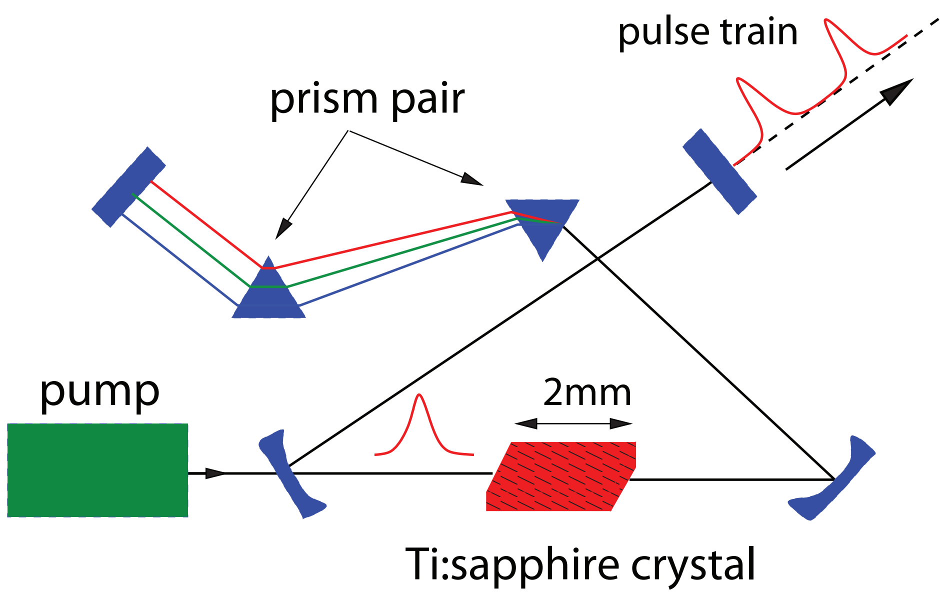

Finally, part of the beam returns to complete the cycle and part is the output and the process is repeated.

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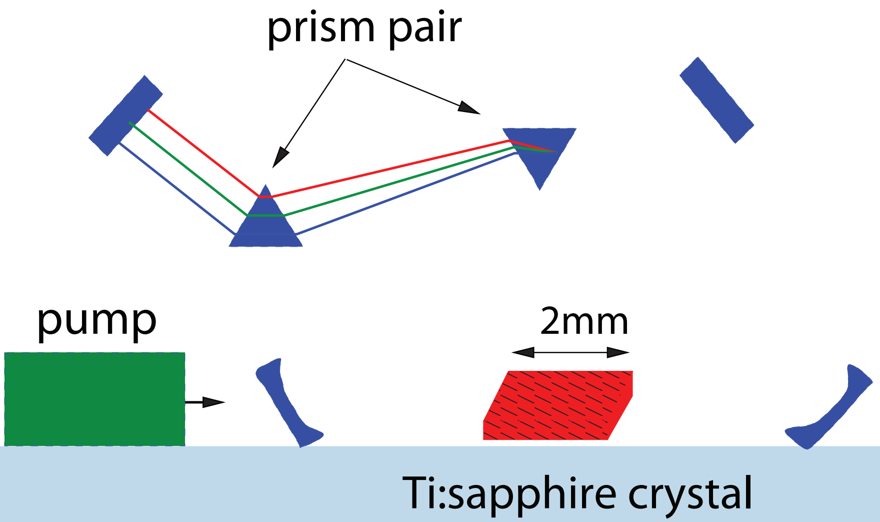

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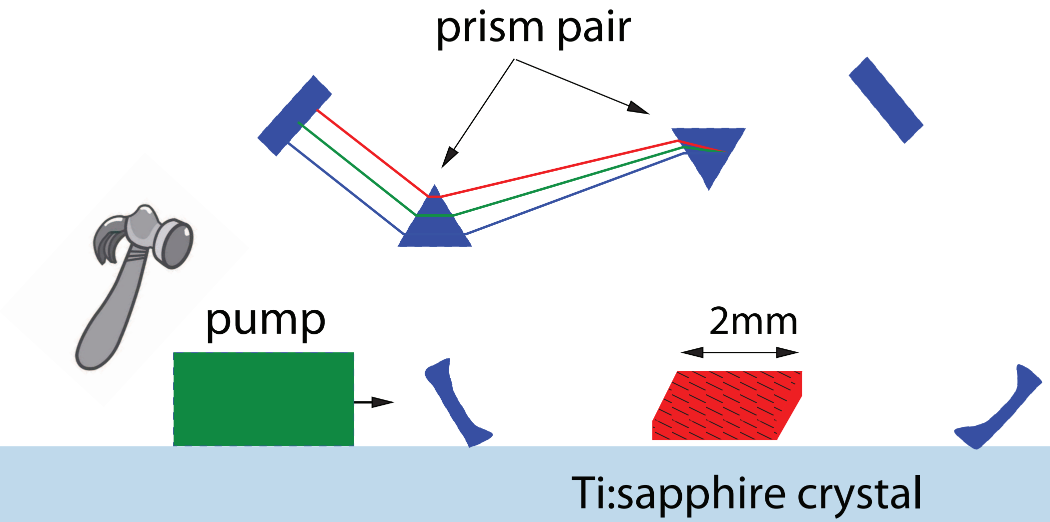

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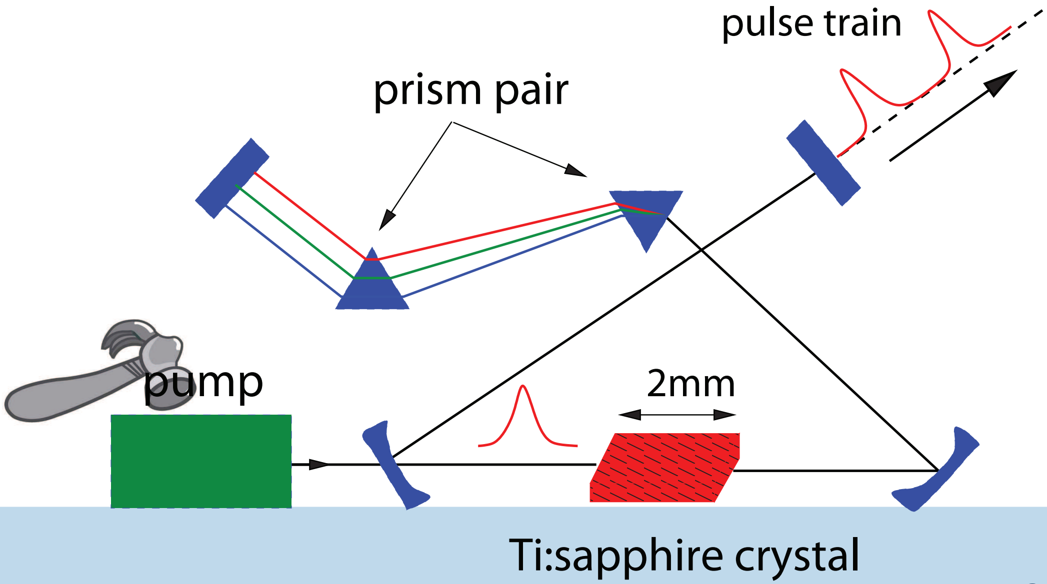

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- Transform the equation into Fourier domain.
- Determine a nonlinear nonlocal integral equation coupled to an algebraic equation.
- This coupling is crucial. It prevents the numerical scheme from diverging.
- Determine the mode from a convergent fixed point iteration scheme.
- The essence of the method is the constant balance between dispersion and nonlinearity.

Spectral Renormalization in detail

For simplicity consider the 2D model:

$$i\frac{\partial \psi}{\partial z} + \nabla^2\psi + N(|\psi|^2)\psi = 0$$

where $\psi = \psi(x, y, z)$, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, and the function $N(|\psi|^2)$ is an arbitrary function of the intensity $|\psi|^2$.

- Look for localized modes: $\psi(x, y, z) = u(x, y)\mathrm{e}^{i\mu z}$, $\mu \in \mathbb{R}$.

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- Define the Fourier transform (FT) of the solution as

$$\hat{u}(\omega_x, \omega_y) = \mathcal{F}\{u\} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u(x,y)\, \mathrm{e}^{i(\omega_x x + \omega_y y)}\, \mathrm{d}x\mathrm{d}y$$

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$$-(\mu + |\boldsymbol{\omega}|^2)\hat{u} + \mathcal{F}\{N(|u|^2)u\} = 0$$

- Introduce the renormalization constant

$$u(x, y) = \lambda v(x, y) \Leftrightarrow \hat{u}(\omega_x, \omega_y) = \lambda \hat{v}(\omega_x, \omega_y)$$

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- Multiply by $\hat{v}^*$ and integrate over the entire space (ω_x, ω_y) to find the algebraic relation

$$-\int_{-\infty}^{+\infty} (\mu + |\boldsymbol{\omega}|^2)|\hat{v}|^2 \, \mathrm{d}\boldsymbol{\omega} + \int_{-\infty}^{+\infty} \mathcal{F}\{N(|\lambda v|^2)v\}\hat{v}^* \, \mathrm{d}\boldsymbol{\omega} = 0$$

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- The solution is obtained by iterating as follows

$$\hat{v}_{n+1}(\boldsymbol{\omega}) = \frac{\mathcal{F}\{N(|\lambda_n v_n|^2)v_n\}}{\mu + |\boldsymbol{\omega}|^2}$$

Final Comments

- This iteration is for $n > 0$. When $n = 0$ an initial guess is required, say a Gaussian or a $\operatorname{sech} x$, that resembles the properties of the required solution.

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- The method converges for arbitrary nonlinearities.
- So far, the method can only converge for the ground states. Higher modes can not be found.

Examples: Vanishing boundary conditions

Consider the 1D nonlinear Schrödinger equation (NLS)

$$i\psi_z + \psi_{tt} + 2|\psi|^2\psi = 0$$
$$\psi(-\infty) = \psi(+\infty) = 0$$

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$$\int_{-\infty}^{+\infty} (\mu + \omega^2)|\hat{v}_n|^2 \mathrm{d}\omega - 2\int_{-\infty}^{+\infty} \mathcal{F}\{|\lambda_n v_n|^2 v_n\}\hat{v}_n^* \,\mathrm{d}\omega = 0$$

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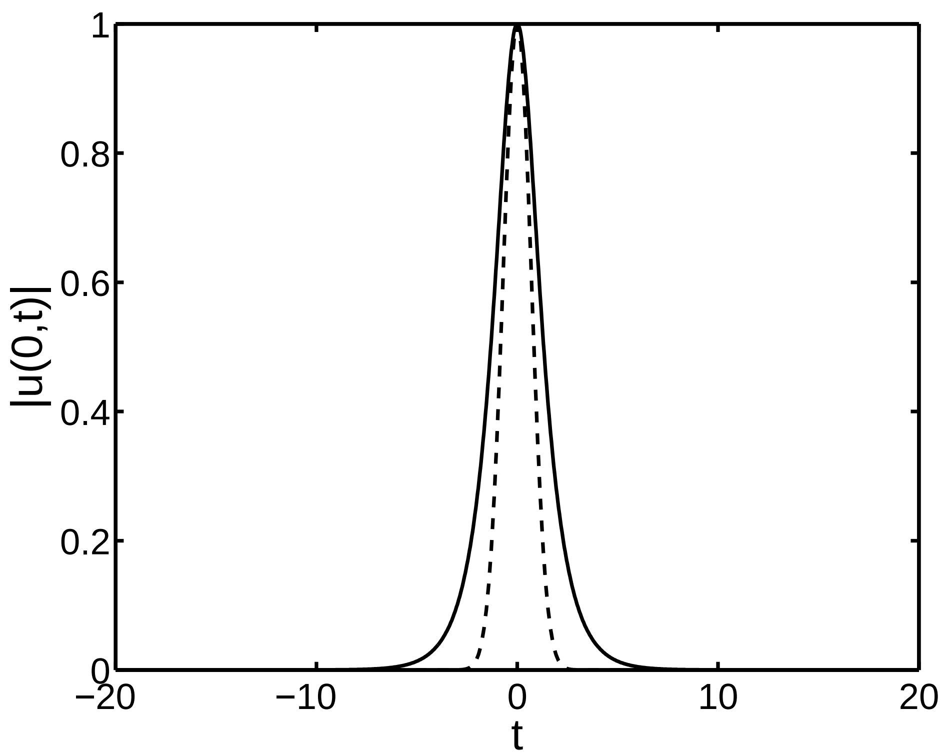
- The iteration scheme is

$$\hat{v}_{n+1} = 2\frac{\mathcal{F}\{|\lambda_n v_n|^2 v_n\}}{\mu + \omega^2}$$

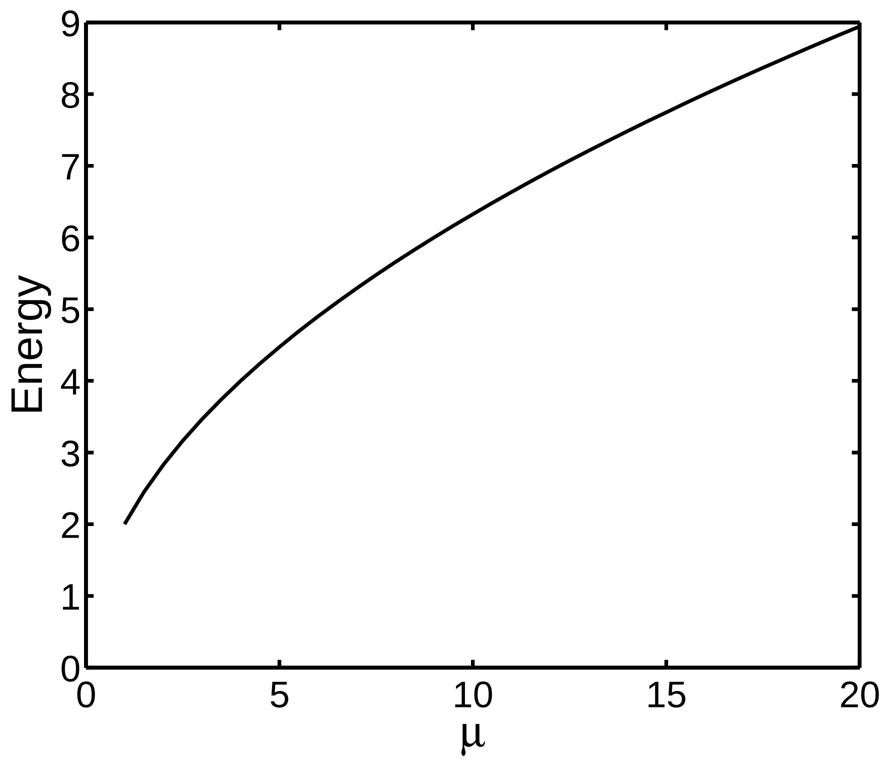
Solution of the NLS

Assuming $v_0 = \exp(-x^2)$ the solution for $\mu = 1$ is:



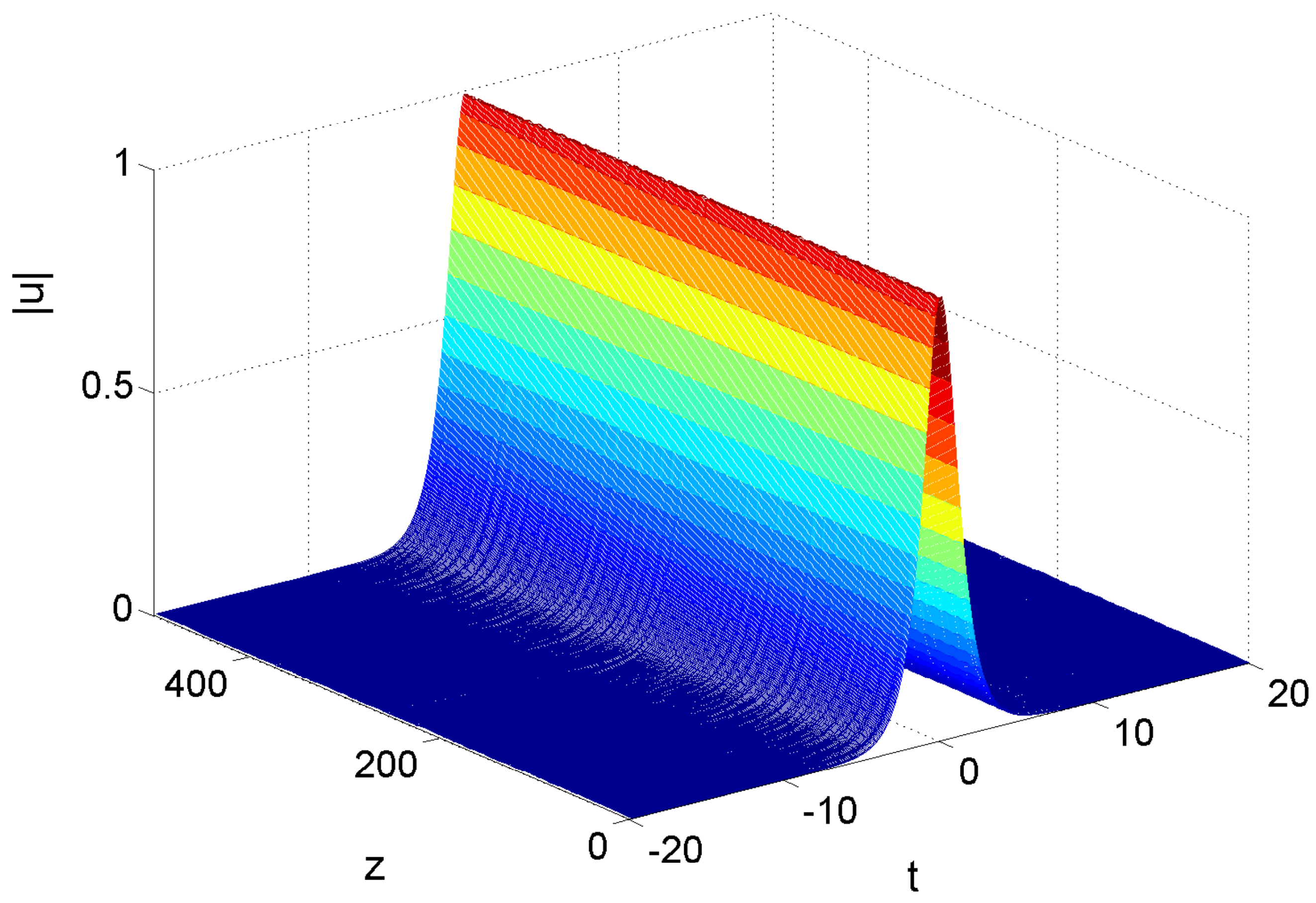
With dashed lines is plotted the initial Gaussian guess.

Solution of the NLS



The energy vs. μ diagram. The analytic result is $E = 2\sqrt{\mu}$ indistinguishable in the graph.

Solution of the NLS



The evolution of the soliton solution as obtained from spectral renormalization.

Saturable nonlinearity

Consider a more interesting system with saturable nonlinearity

$$i\psi_z + \frac{1}{2}\psi_{tt} - \frac{1}{1+|\psi|^2}\psi = 0$$

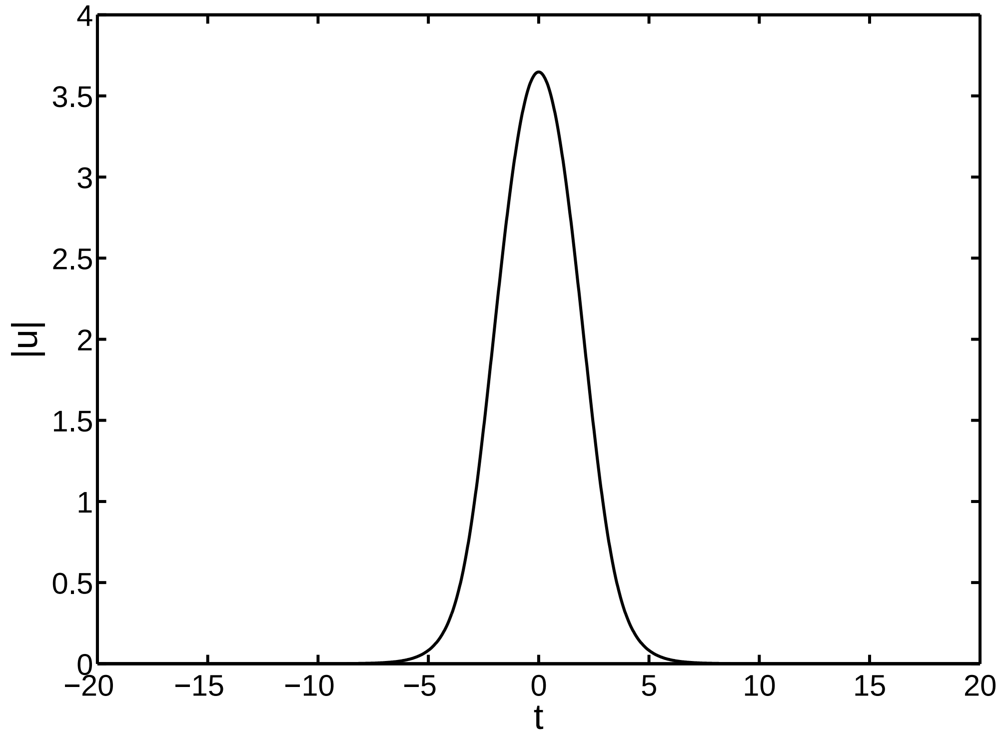
$$\psi(-\infty) = \psi(+\infty) = 0$$

Taking solutions of the form $\psi(z,t) = u(t)\mathrm{e}^{-i\mu z}$ the iteration scheme is

$$\hat{v}_{n+1} = \frac{\mu + r}{r + \omega^2/2}\hat{v}_n - \frac{1}{r+\omega^2/2}\mathcal{F}\left\{\frac{v_n}{1+|\lambda_n v_n|^2}v_n\right\}$$

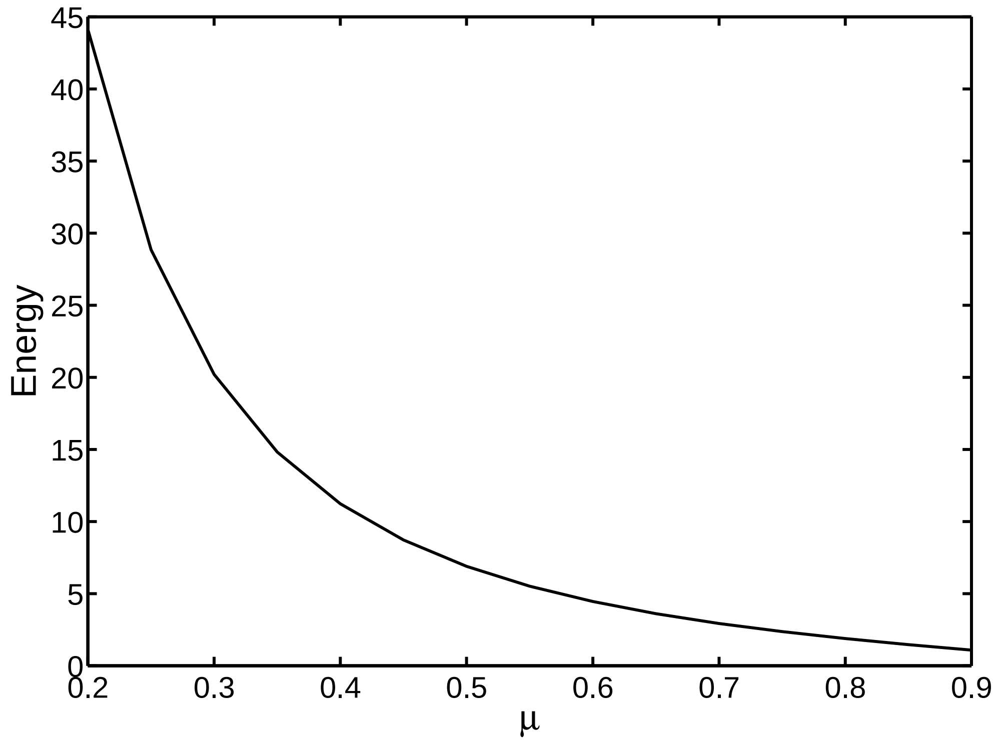
$$\int_{-\infty}^{+\infty}(\mu - \omega^2/2)|\hat{v}_n|^2\,\mathrm{d}\omega - \int_{-\infty}^{+\infty}\mathcal{F}\left\{\frac{v_n}{1+|\lambda_n v_n|^2}\right\}\hat{v}_n^*\,\mathrm{d}\omega = 0$$

The soliton solution



Saturable nonlinearity

and the energy vs. μ graph



Nonvanishing boundary conditions

Let us consider the NLS equation in the normal regime,

$$i\psi_z - \frac{1}{2}\psi_{tt} + |\psi|^2\psi = 0$$

We need to seek solutions of the form

$$\psi(z,t) = u(t)\mathrm{e}^{i\mu z + i\phi(t)}$$

where $u(t)$ and $\phi(t)$ are now real functions of t. Separate real and imaginary parts to finally obtain for the amplitude

$$-\mu u - \frac{1}{2}u_{tt} + \frac{A^2}{2u^3} + u^3 = 0$$

where A is a constant.

Nonvanishing boundary conditions

Differentiate and set $u_t = v$ to obtain the equation for v

$$-\mu v - \frac{1}{2} v_{tt} - \frac{3A^2}{2u^4} v + 3u^2 v = 0$$

The iteration scheme is

$$v = \lambda w, \quad u_n = \lambda_n \left(\int w_n \, \mathrm{d}t + u_\infty \right)$$

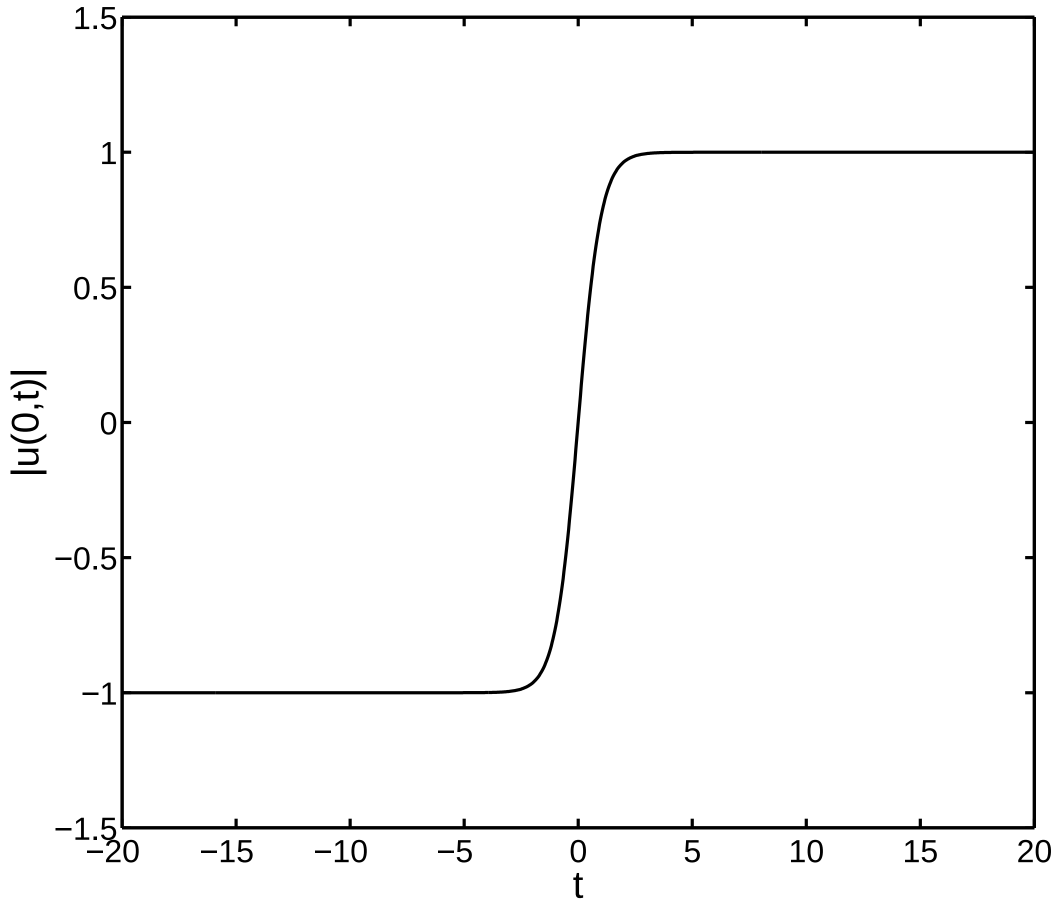
$$\int_{-\infty}^{+\infty} (-\mu + \omega^2) |\hat{w_n}|^2 \, \mathrm{d}\omega -$$

$$\int_{-\infty}^{+\infty} \mathcal{F} \left\{ \frac{3A^2}{2\lambda_n^4 u_n^4} w_n - 3\lambda_n^2 u_n^2 w_n \right\} \hat{w_n}^* \, \mathrm{d}\omega = 0$$

$$\hat{w}_{n+1} = \frac{\mu + r}{r + \omega^2/2} \hat{w}_n + \frac{\mathcal{F}\left\{ \frac{3A^2}{2\lambda^4 u_n^4} w_n - 3\lambda^2 u_n^2 w_n \right\}}{r + \omega^2/2}$$

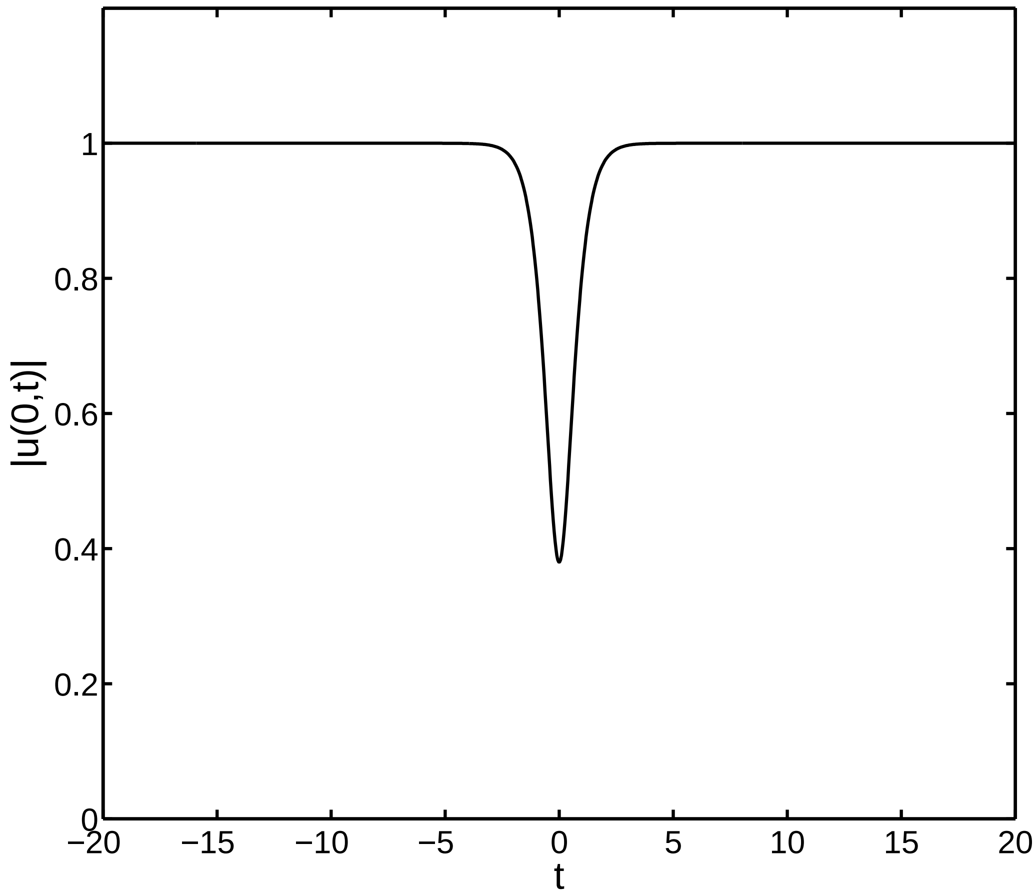
Nonvanishing boundary conditions

With $A = 0$ we get black solitons



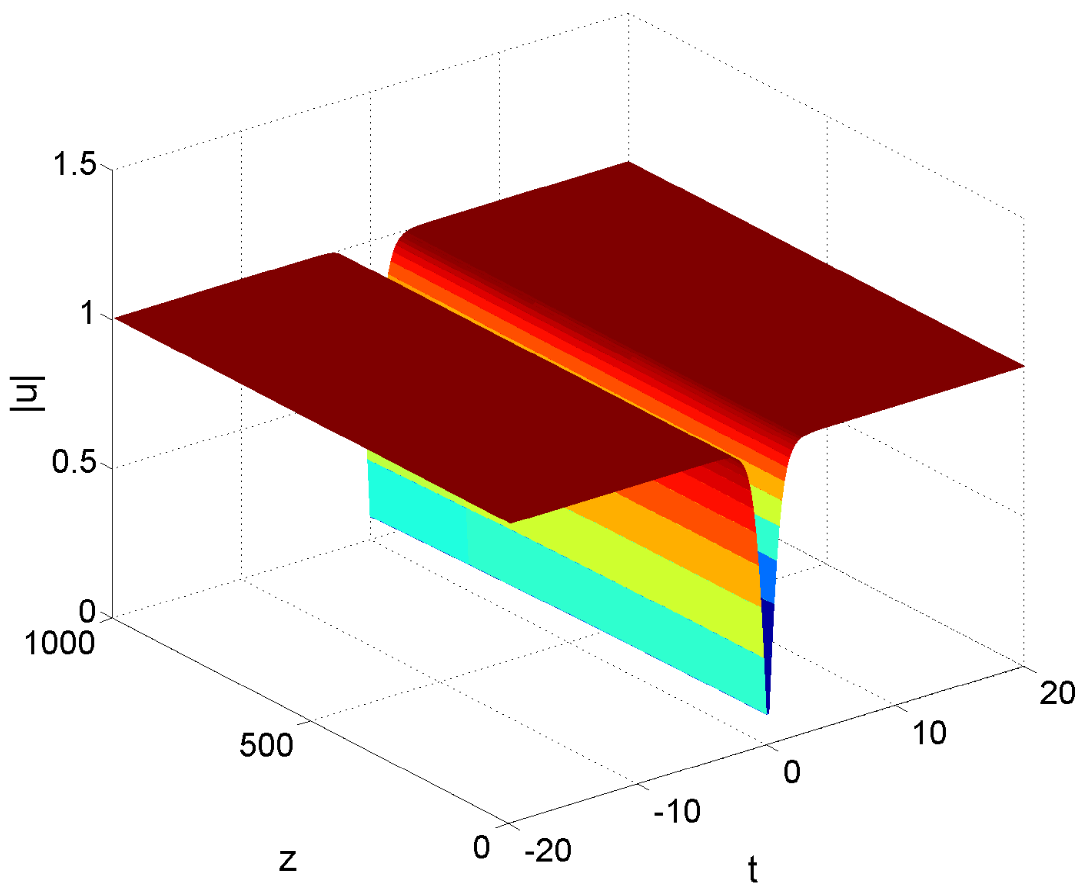
Nonvanishing boundary conditions

with $A \neq 0$ we get gray



Nonvanishing boundary conditions

The evolution of a dark soliton



The master equation

Haus was the first to derive this equation to model mode–locked lasers:

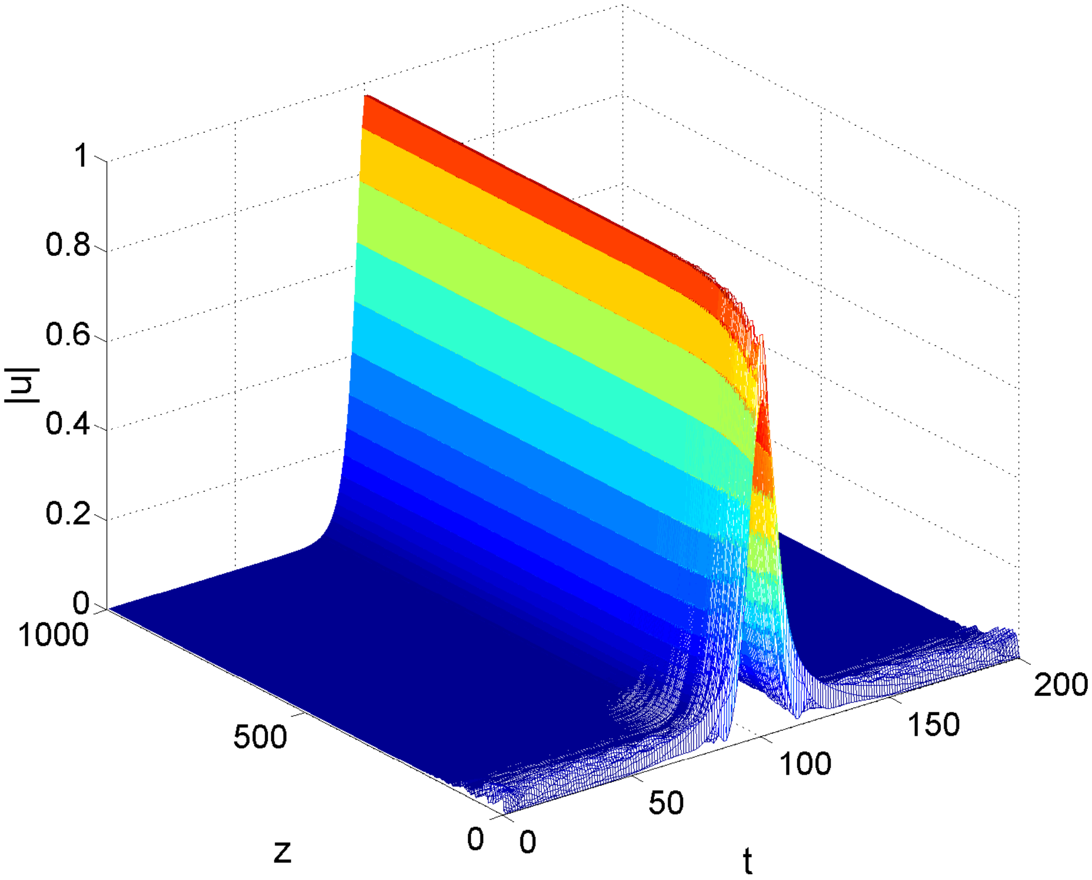
$$i\psi_z + (1 - i\tau g(z))\psi_{tt} + (4 - i\beta)|\psi|^2\psi + i(\gamma - g(z))\psi = 0$$

where

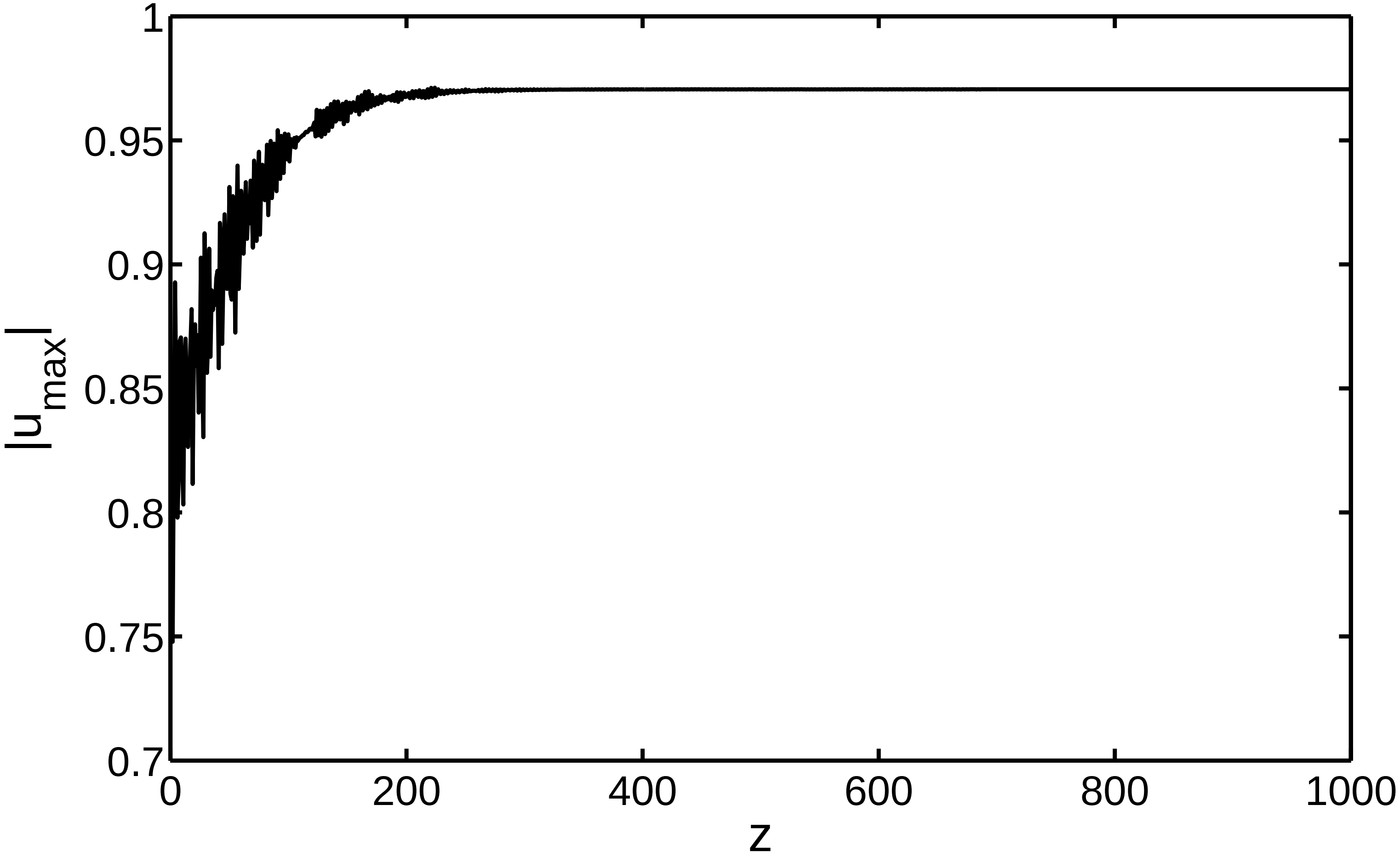
$$g(z) = \frac{2g_0}{1 + \int_{-\infty}^{+\infty} |\psi|^2 \, \mathrm{d}t / e_0}$$

The dynamics, solutions and their stability crucially depend on the values of g_0, e_0, γ and β.
A perturbative analysis and a special set of solutions for $e_0 = \int_{-\infty}^{+\infty} |\psi|^2 \, \mathrm{d}t$ was given by Kutz.
Hereafter, $e_0 = 1$, and $g_0 = \tau = \gamma = 0.1$ and we study the equation for the different values of β.

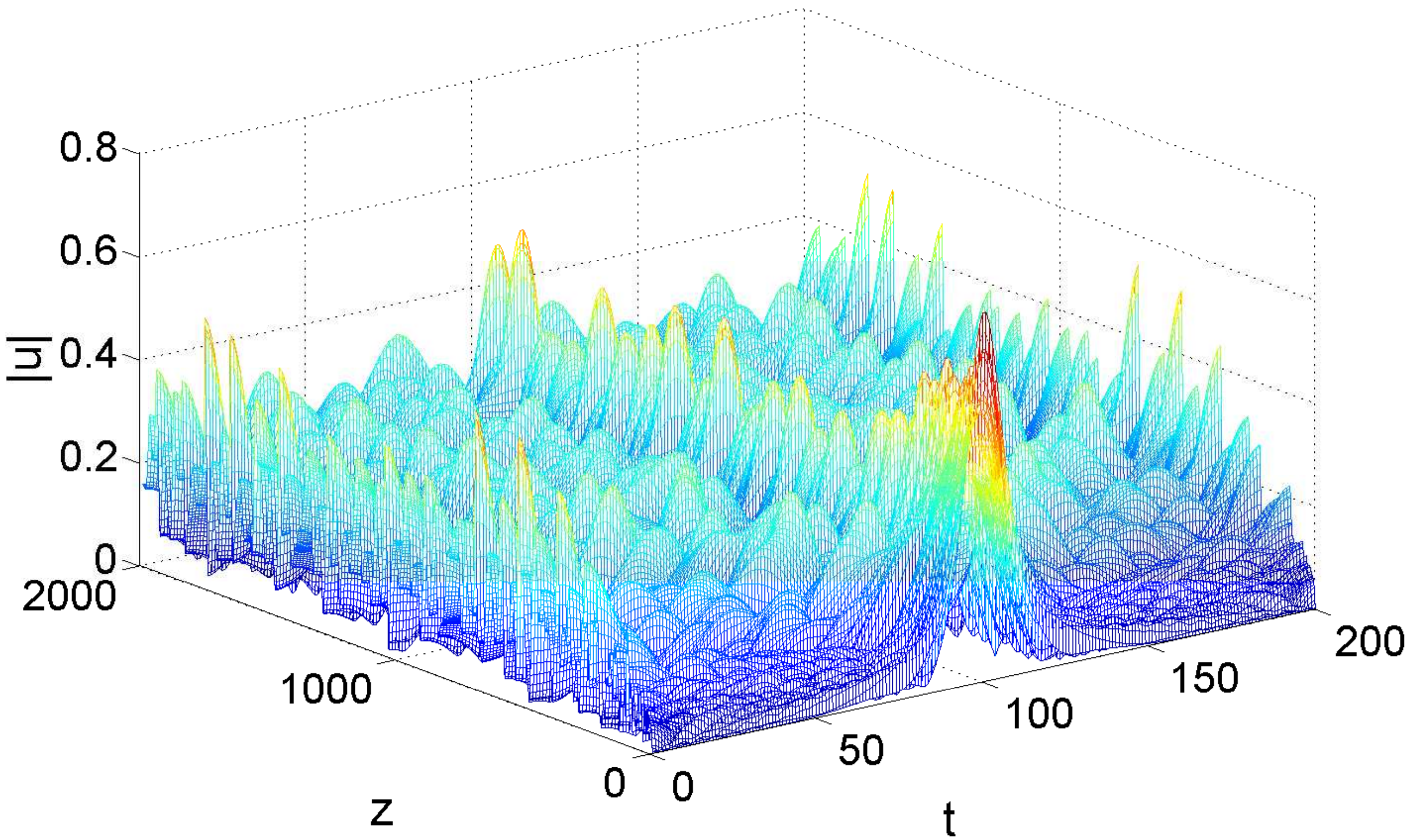
Stable pulse for $0.01 < \beta < 0.0348$



Evolution of the soliton peak for $0.01 < \beta < 0.0348$

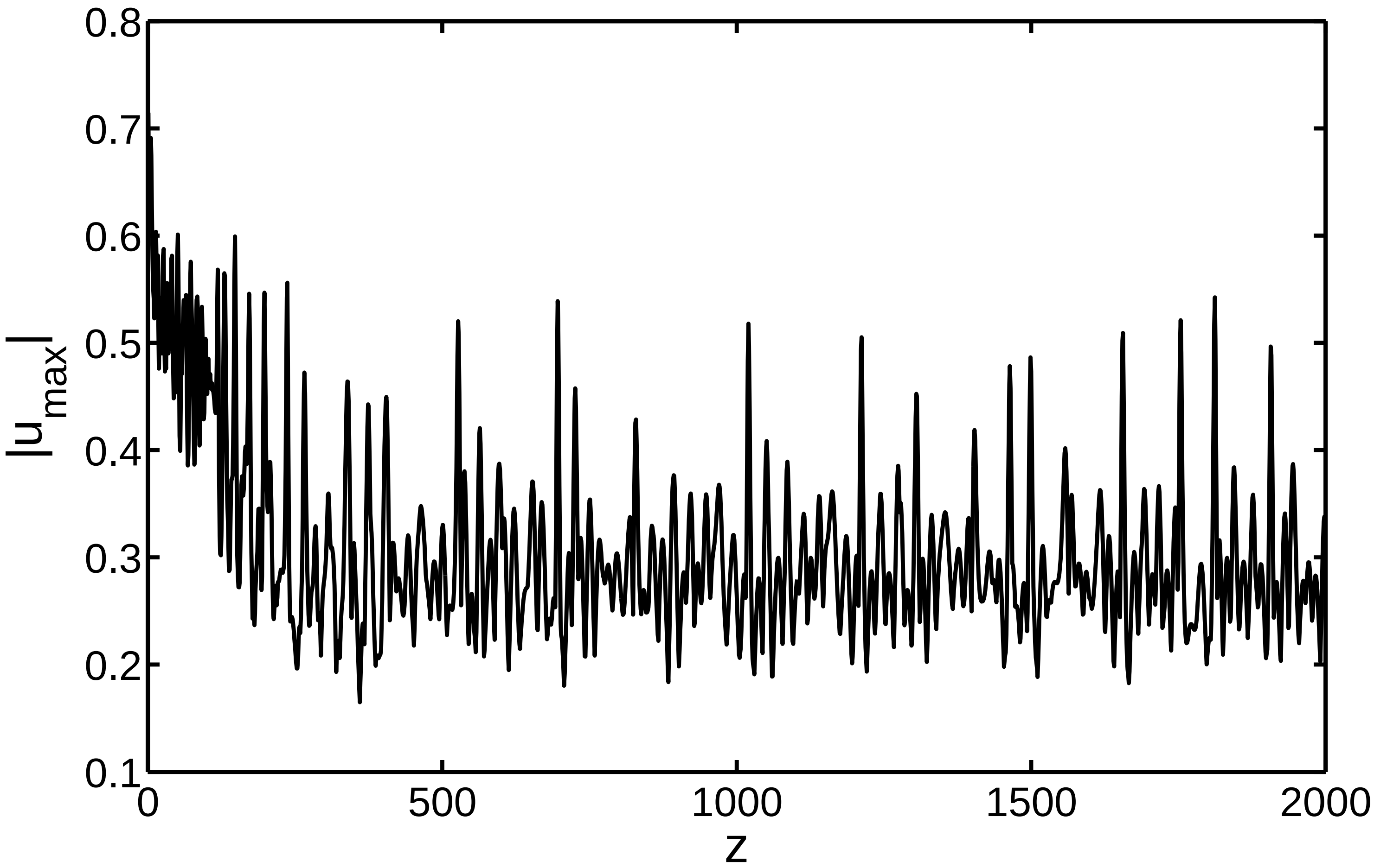


Unstable pulse for $\beta < 0$

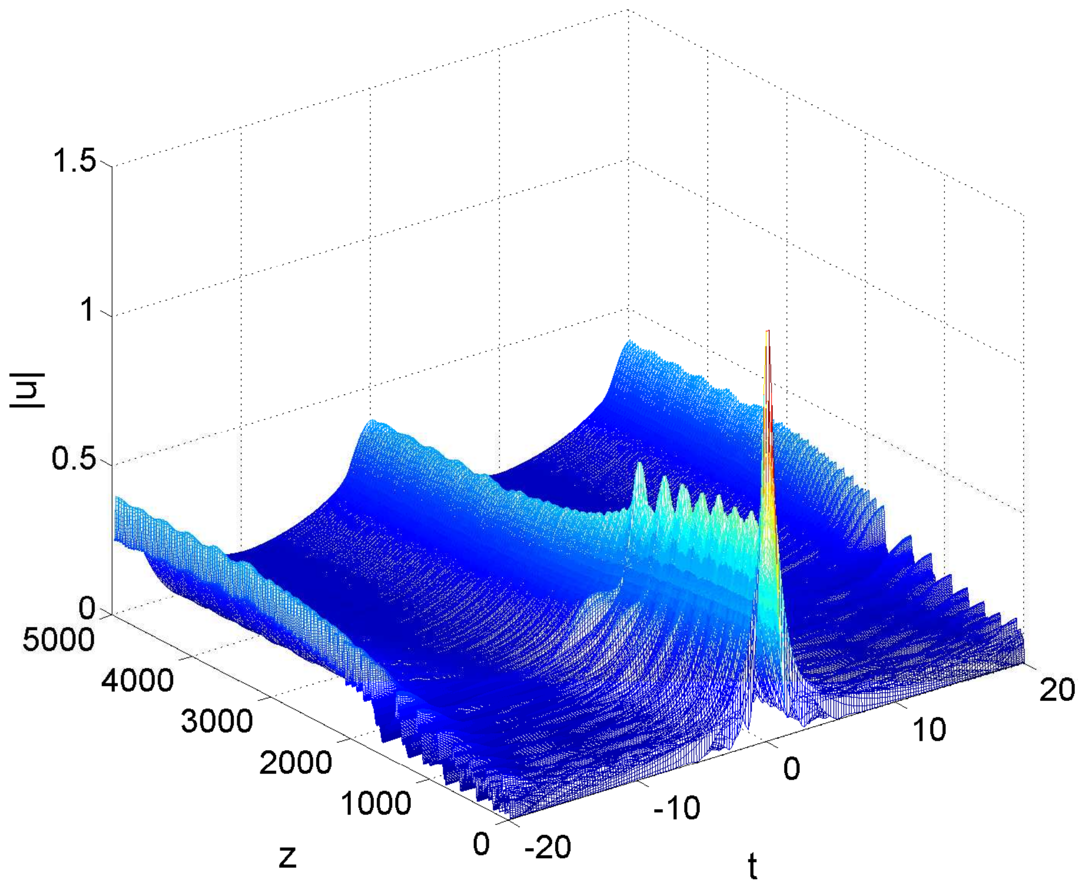


Dynamics of the master equation

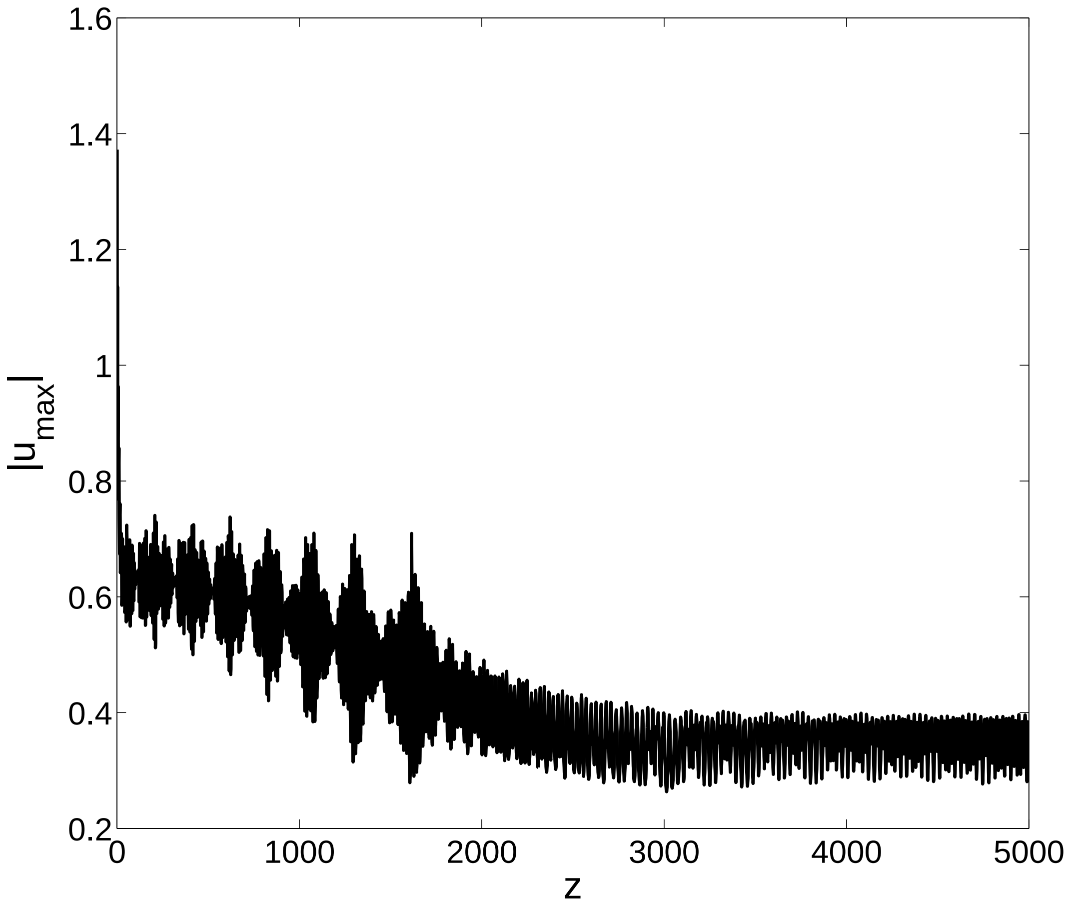
Evolution of the soliton peak for $\beta < 0$



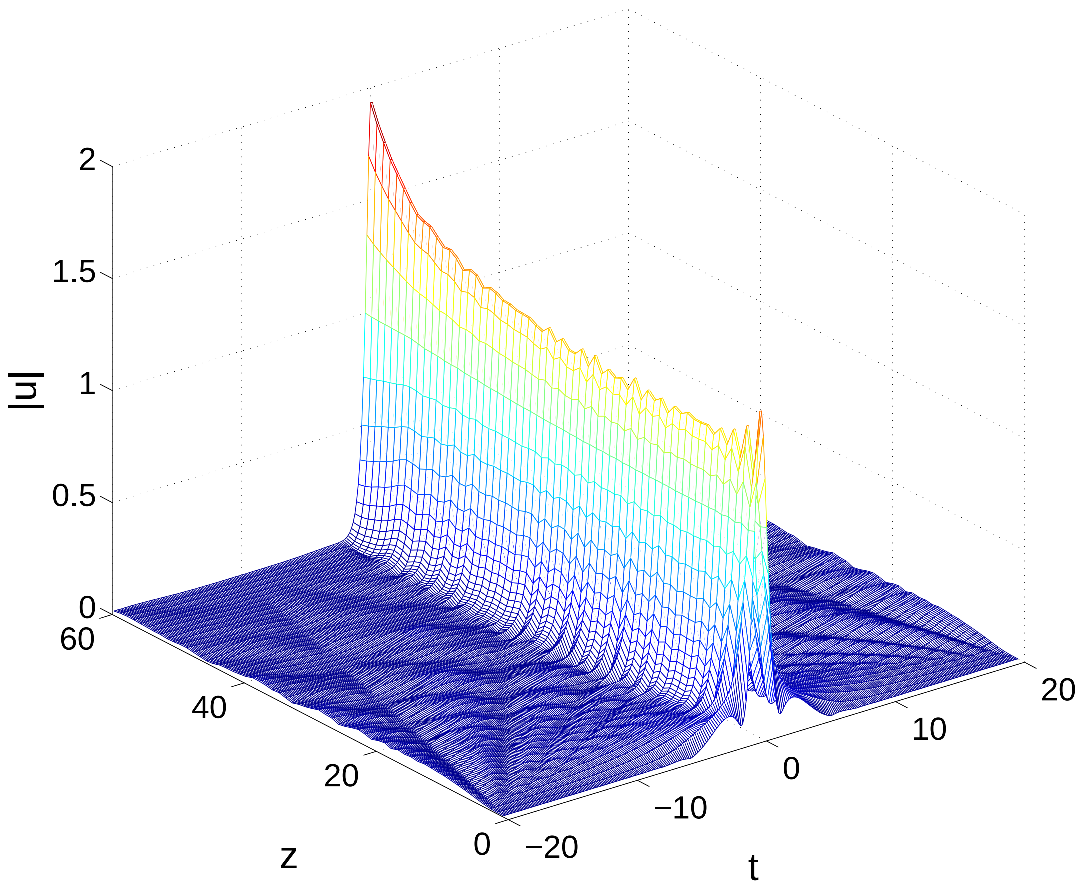
Quasi–periodic evolution for $0 < \beta < 0.01$



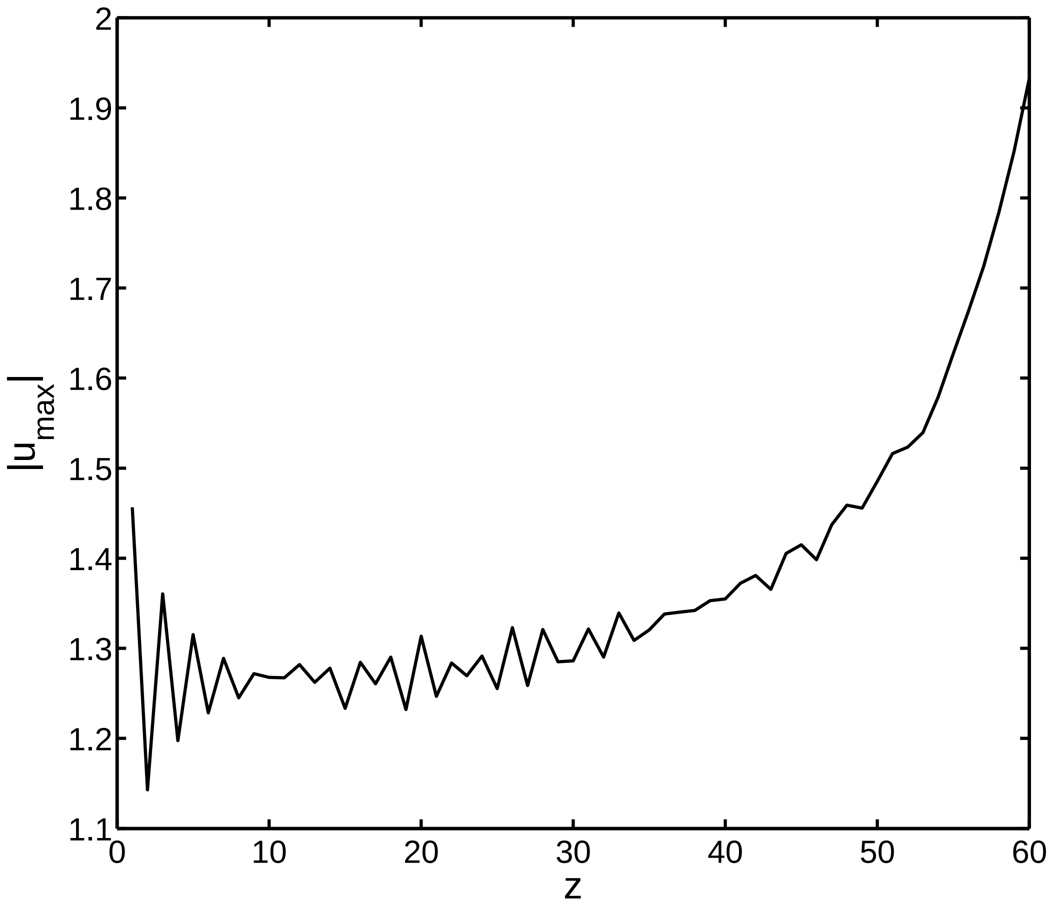
Evolution of the soliton peak for $0 < \beta < 0.01$



Blow–up occurs for $\beta > 0.0348$



Evolution of the soliton peak for $\beta > 0.0348$



Solutions of the master equation

Using spectral renormalization we derive the following iteration scheme:

$$\hat{v}_{n+1} = \frac{\mathcal{F}\{-i\tau G_n v_{n,tt} + (4 - i\beta)|\lambda_n v_n|^2 v_n - iG_n v_n\}}{\mu + \omega^2 - i\gamma}$$

$$\int_{-\infty}^{+\infty} (\mu + \omega^2 - i\gamma)|\hat{v}_n|^2 \, \mathrm{d}\omega =$$

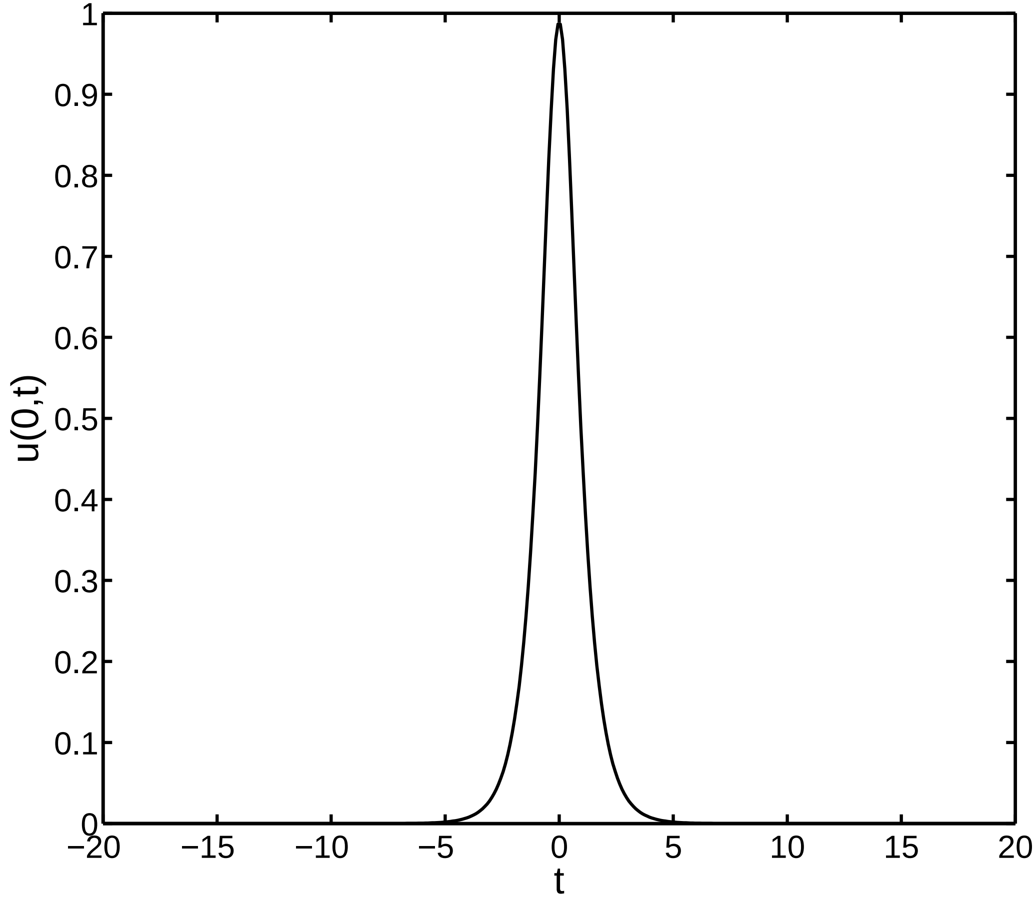
$$\int_{-\infty}^{+\infty} \mathcal{F}\{-i\tau G_n v_{n,tt} + (4 - i\beta)|\lambda_n v_n|^2 v_n - iG_n v_n\}\hat{v}_n^* \, \mathrm{d}\omega$$

where

$$G_n = \frac{2g_0}{1 + \int_{-\infty}^{+\infty} |\lambda_n v_n|^2 \, \mathrm{d}t/e_0}$$

Modes, ie solutions to the equation are found for specific values of μ.

Stable pulse for $\beta = 0.034$ and $\mu = 1.955$



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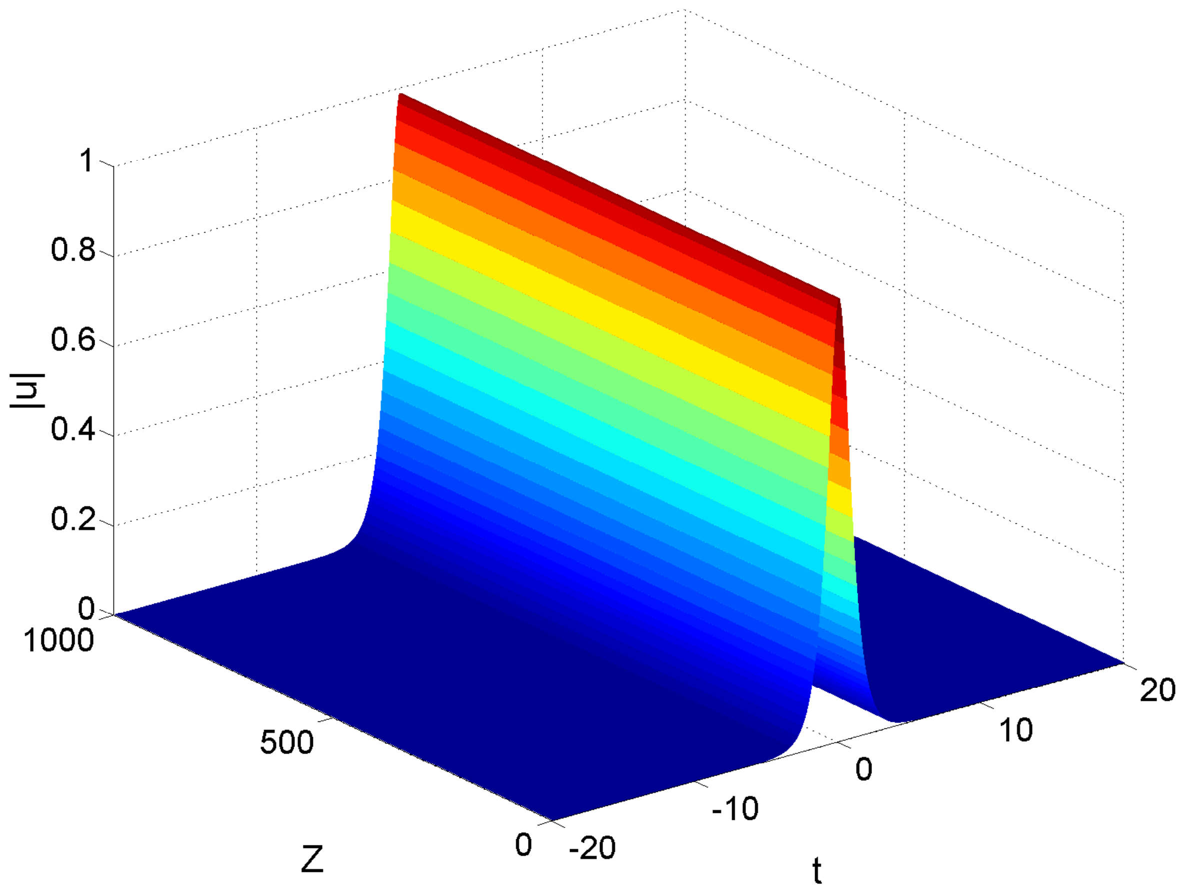

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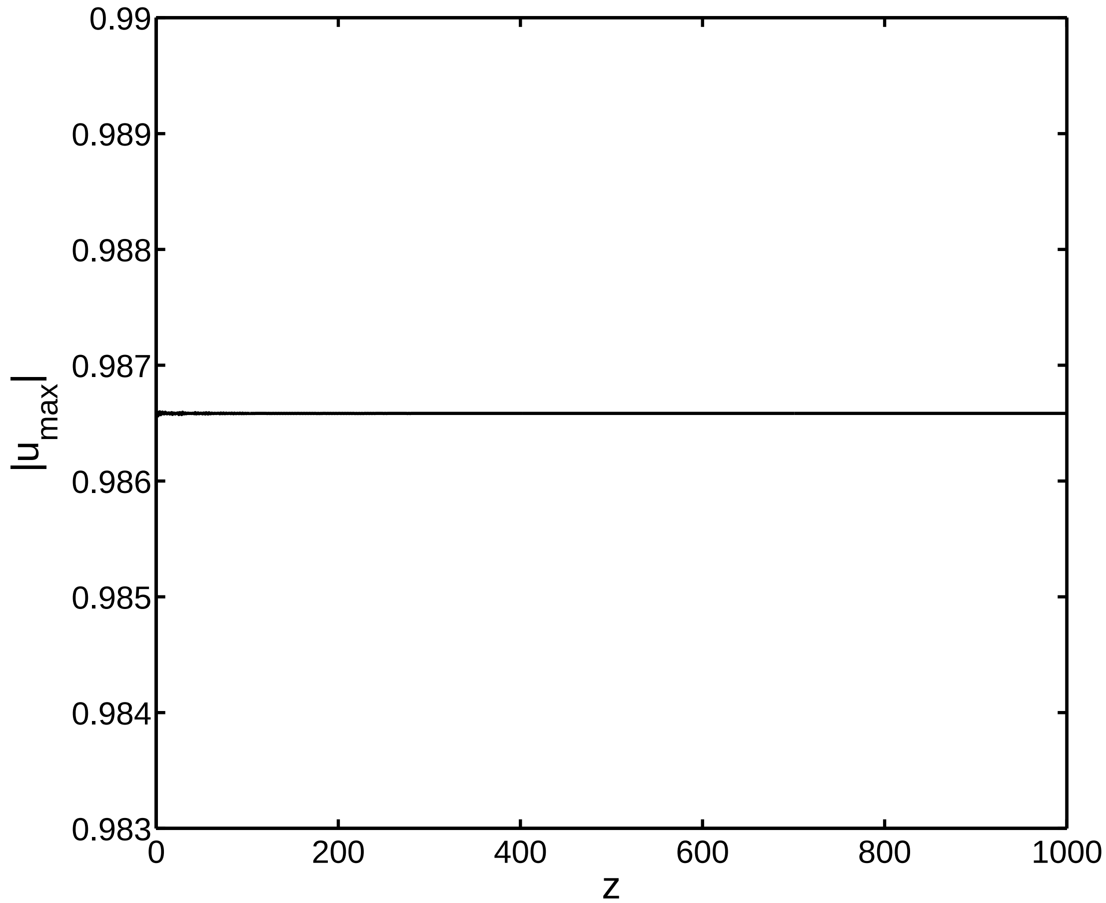
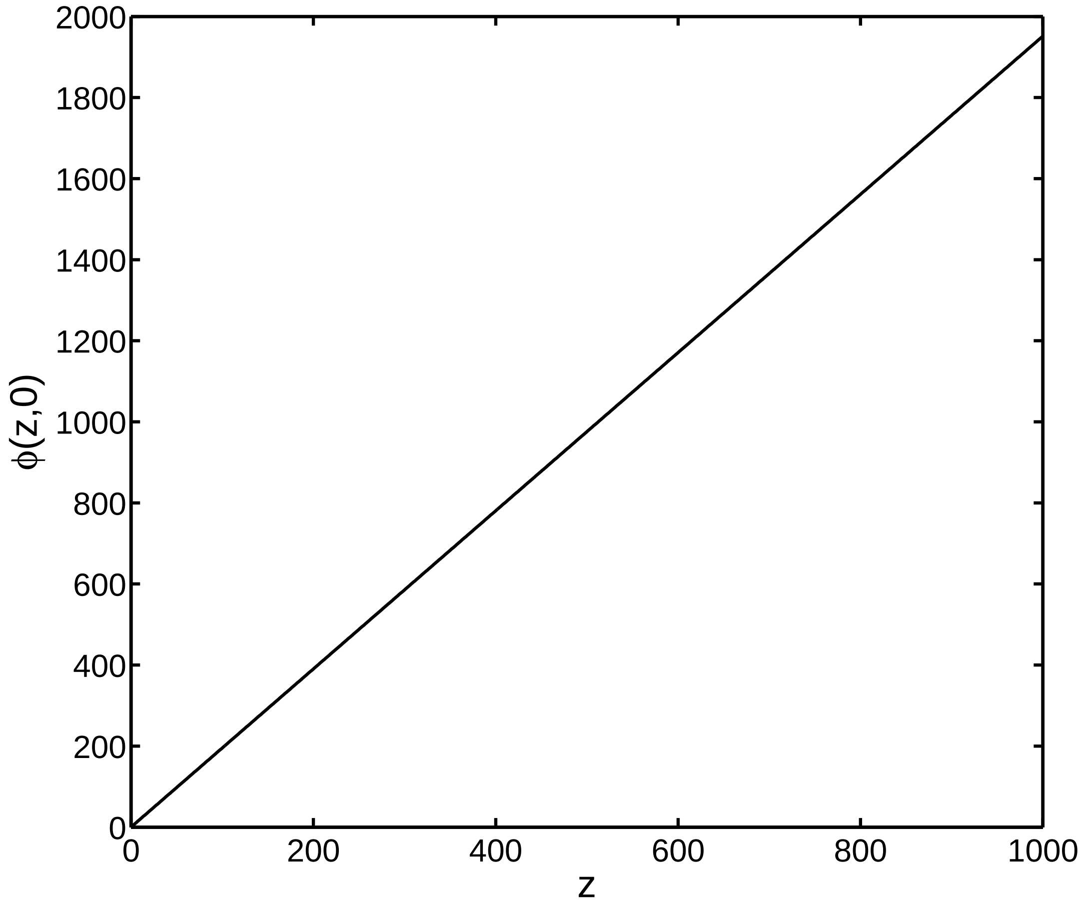
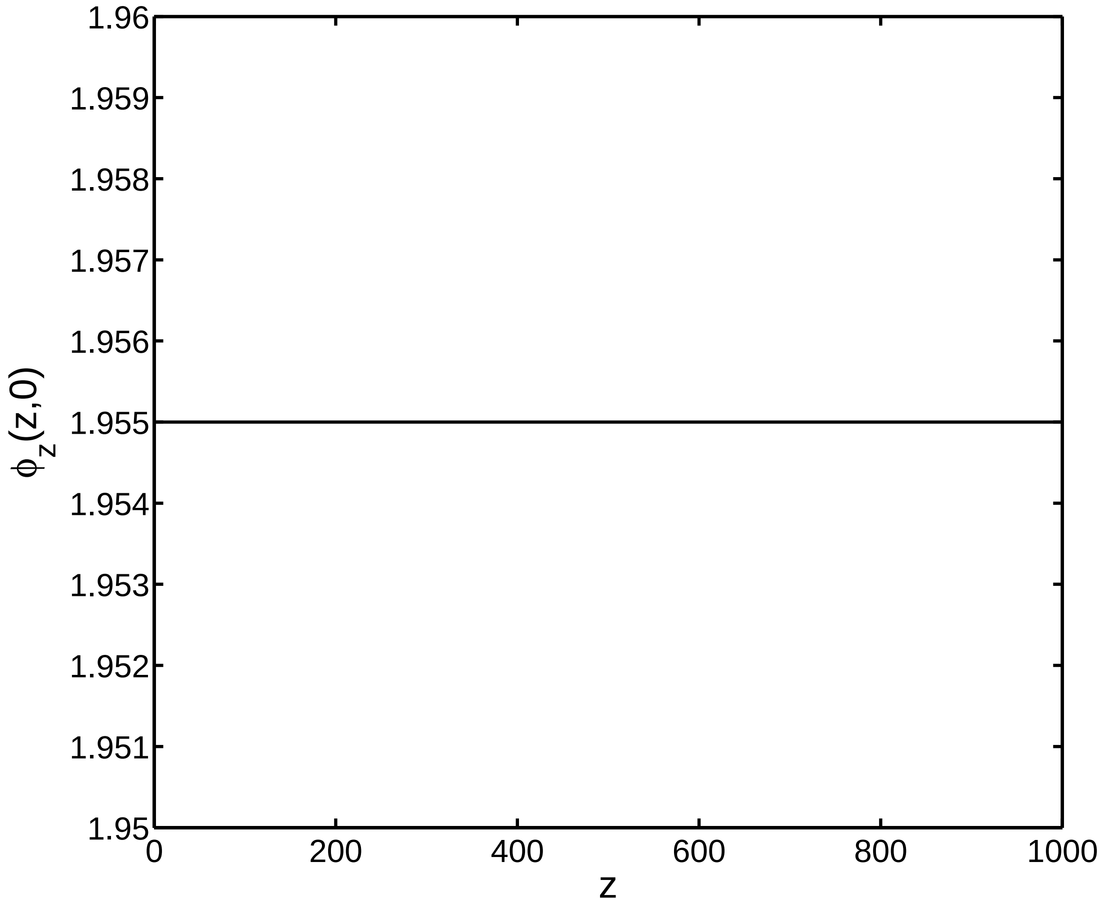


Evolution of the soliton phase $\psi(z)$ for $\beta = 0.034$ and $\mu = 1.955$

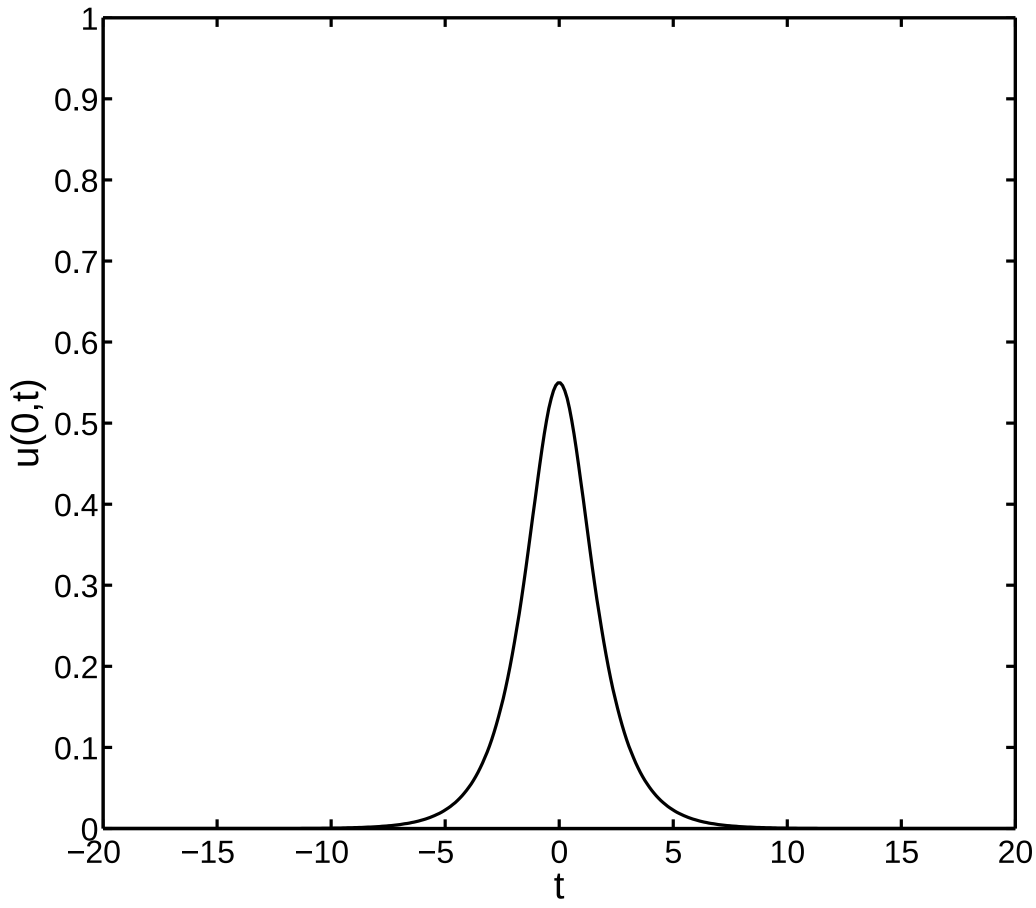


Evolution of $\phi_z(=\mu)$ for $\beta = 0.034$ and $\mu = 1.955$

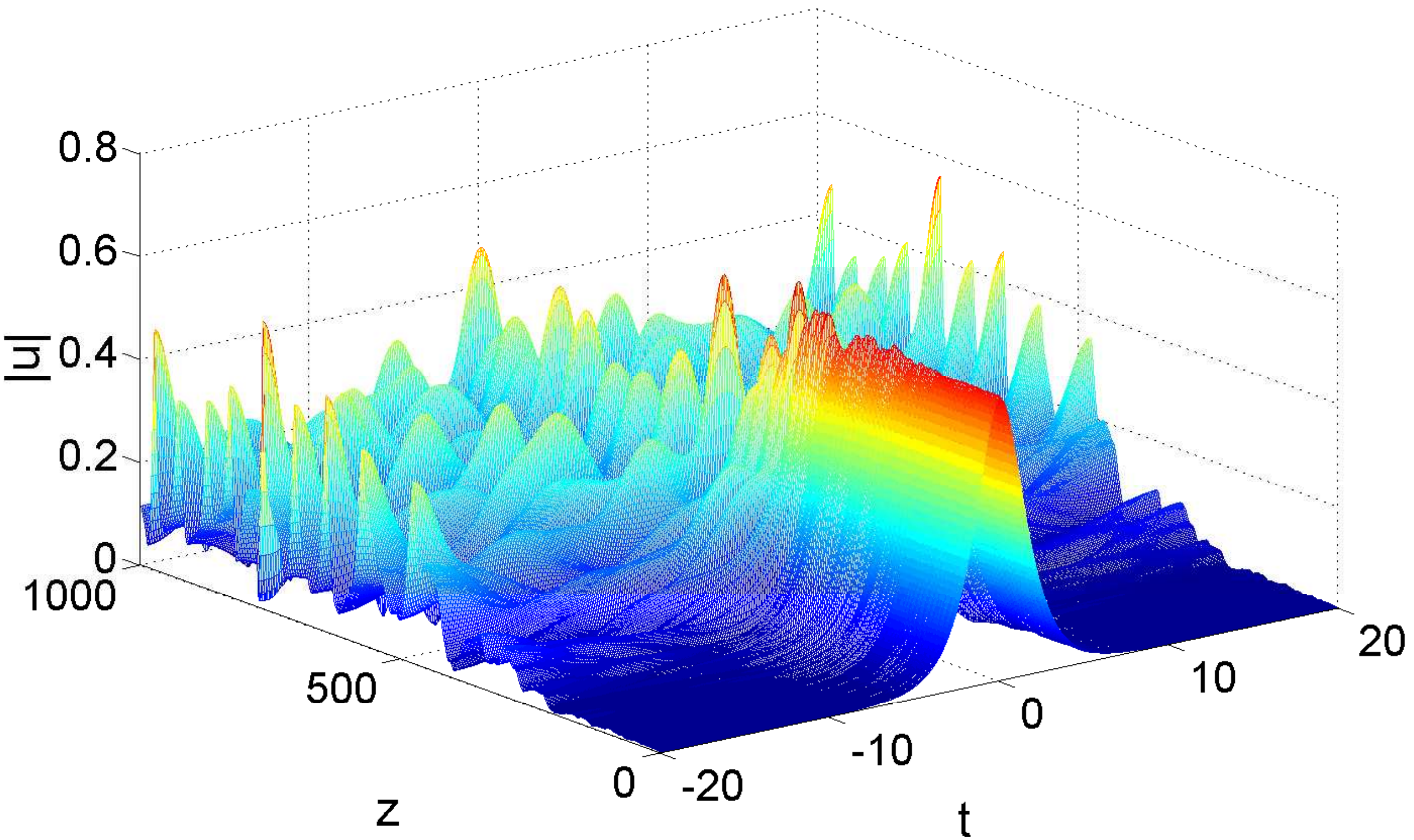


Solutions of the master equation

Unstable pulse for $\beta = -0.05$ and $\mu = 0.605$



Evolution of the pulse for $\beta = -0.05$ and $\mu = 0.605$



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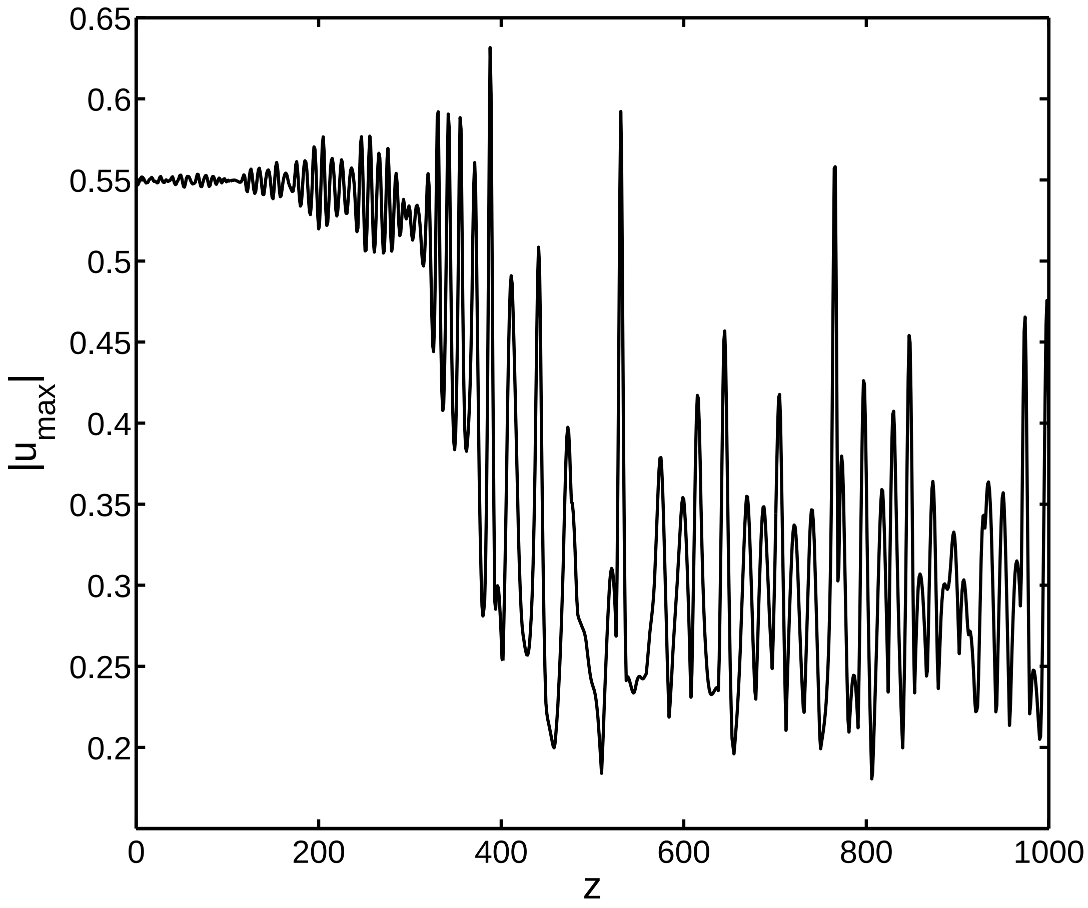
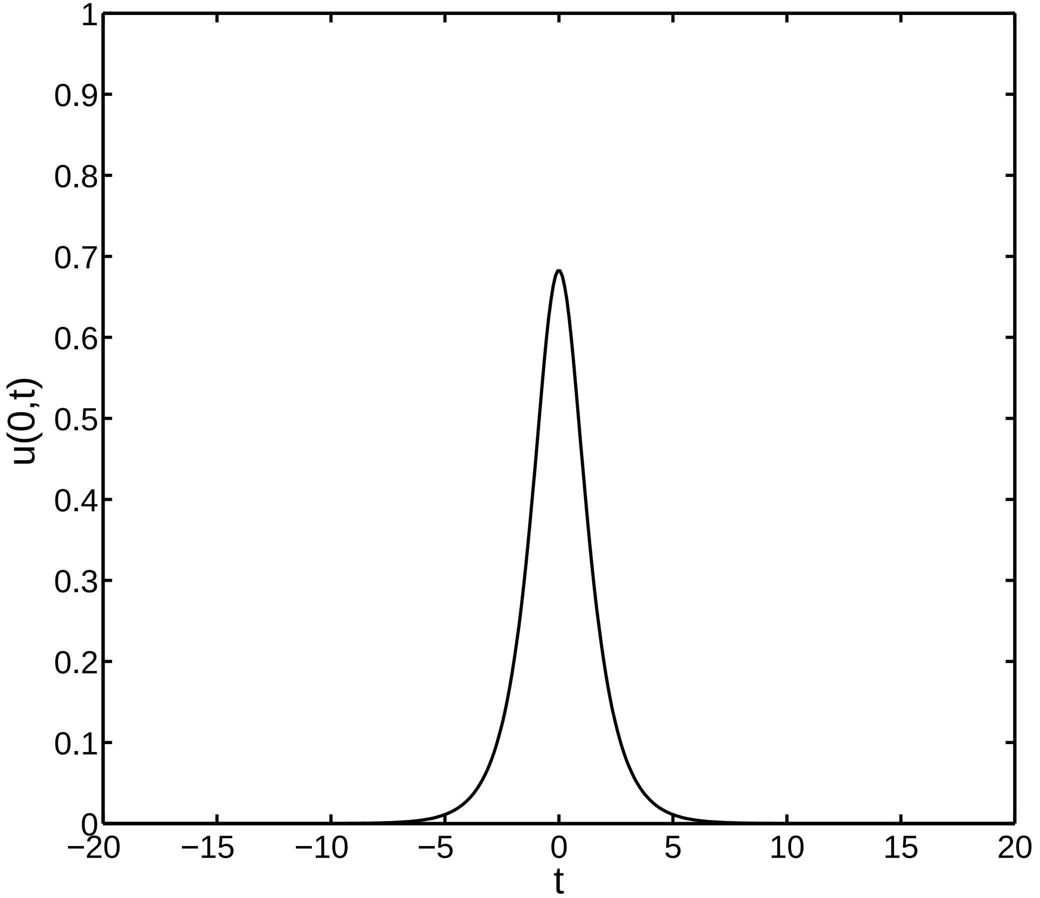
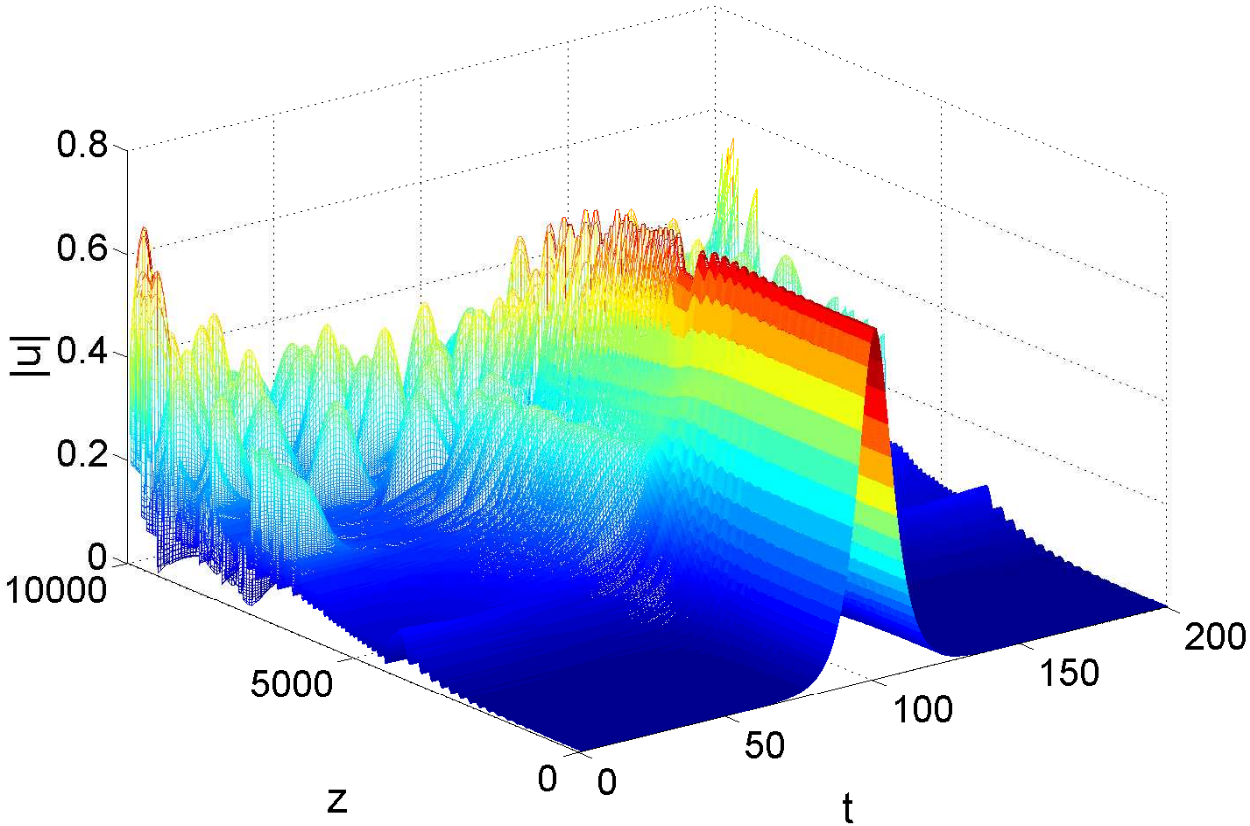


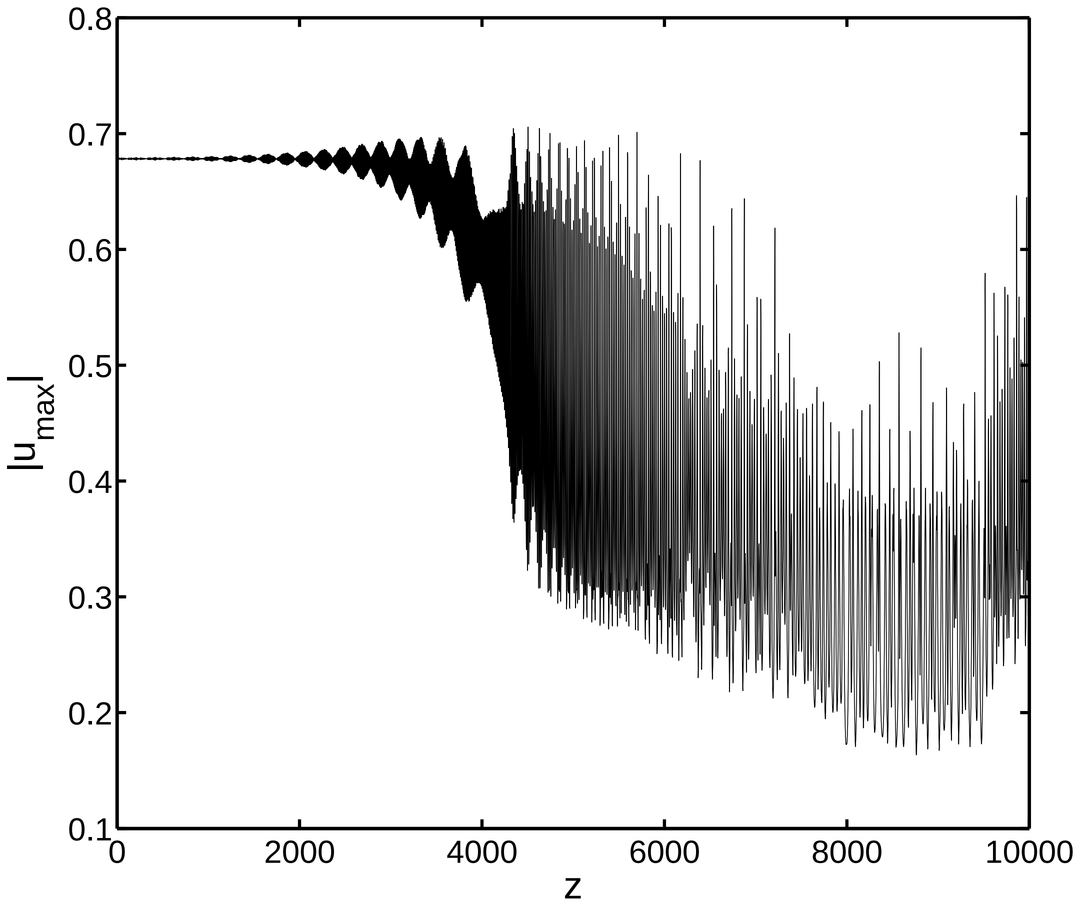
The pulse for $\beta = 0.005$ and $\mu = 0.933$



Quasi–periodic solution of the pulse for $\beta = 0.005$ and $\mu = 0.933$



Evolution of the soliton peak for $\beta = 0.005$ and $\mu = 0.933$



Summary

- The master equation is a phenomenological model that describes pulse propagation in a laser cavity.

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- The master equation is a phenomenological model that describes pulse propagation in a laser cavity.
- Mode–locking only occurs for a narrow range of parameters.
- Unstable pulses exist.
- Blow–up may occur in the evolution of an arbitrary pulse. Modes that blow–up do not exist.

The mode–locking equation

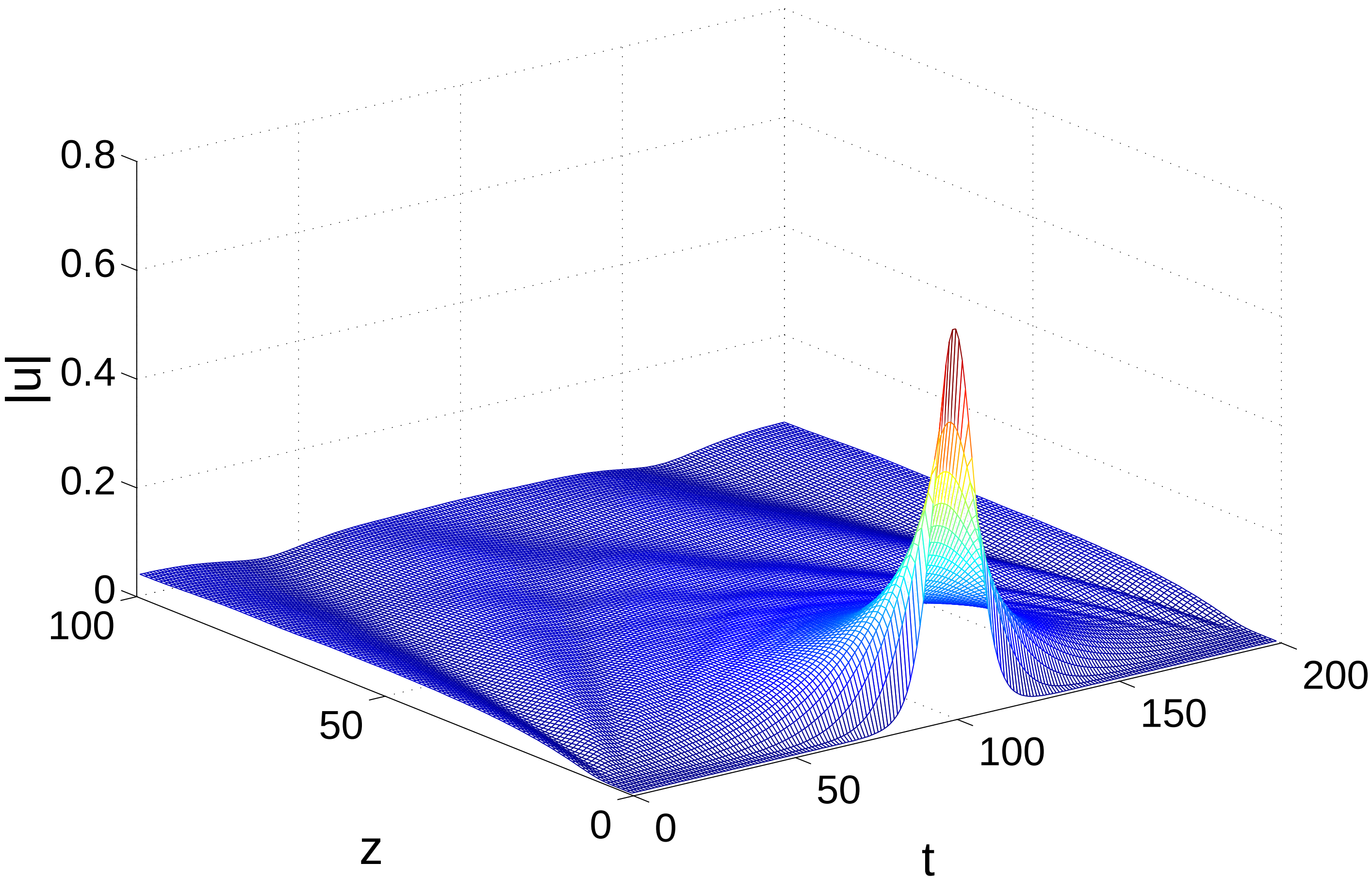
Let us now consider the following model for the propagation of pulses in a laser cavity

$$i\psi_z + \frac{1}{2}\psi_{tt} + |\psi|^2\psi =$$

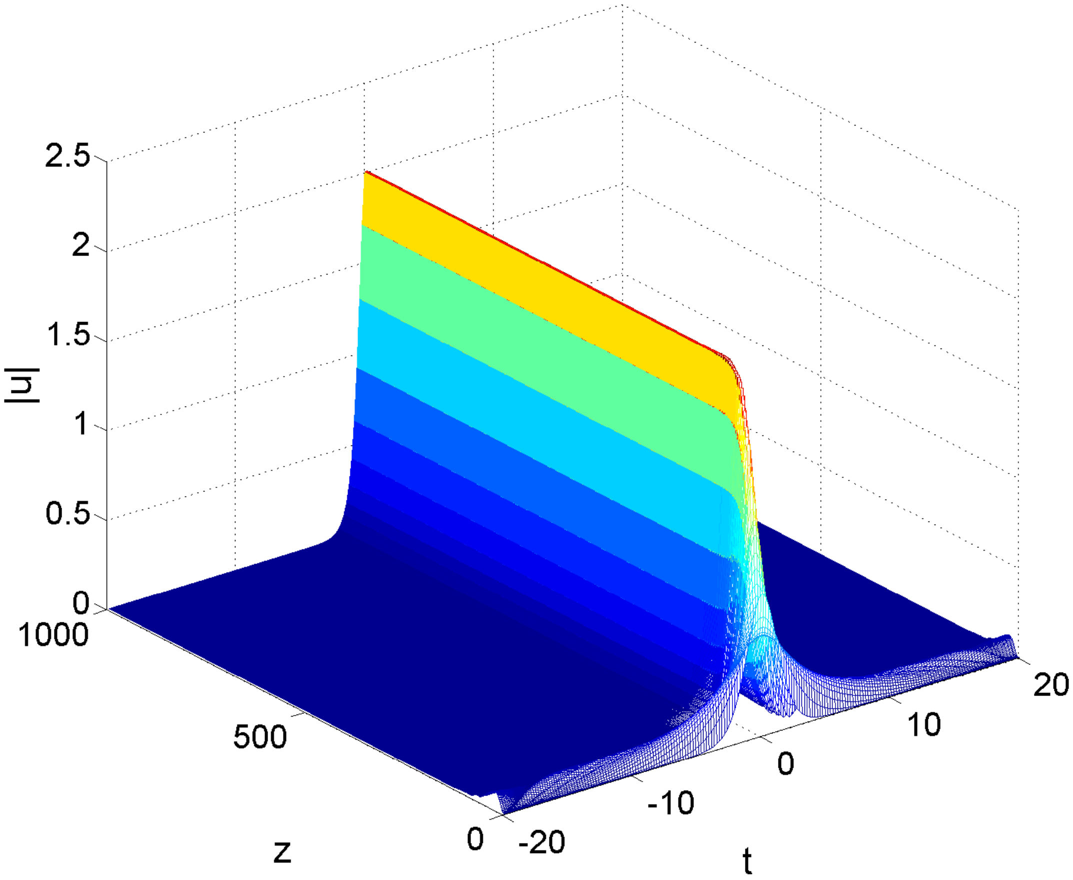
$$\frac{ig}{1+\epsilon\int_{-\infty}^{+\infty}|\psi|^2\,\mathrm{d}t}\psi + \frac{i\tau}{1+\gamma\int_{-\infty}^{+\infty}|\psi|^2\,\mathrm{d}t}\psi_{tt} - \frac{il}{1+\delta|\psi|^2}\psi$$

where the parameters g, τ, l, ϵ, γ and δ are all positive, real constants. The first term on the right hand side represents saturable gain, the second is nonlinear filtering and the third saturable loss.
In our analysis we will fix $\tau = l = 0.1$ and $\epsilon = \gamma = \delta = 1$ and we will modify the gain parameter g.

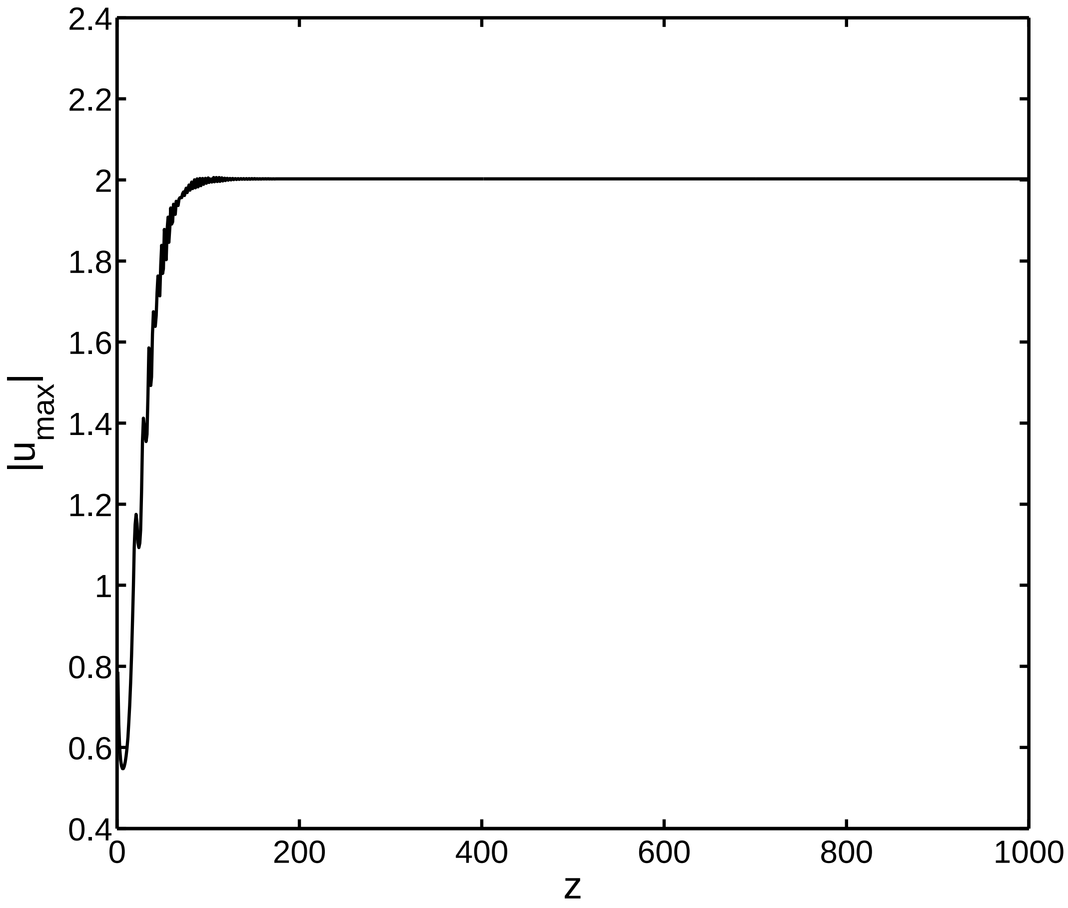
Unstable pulse for $g = 0.1$



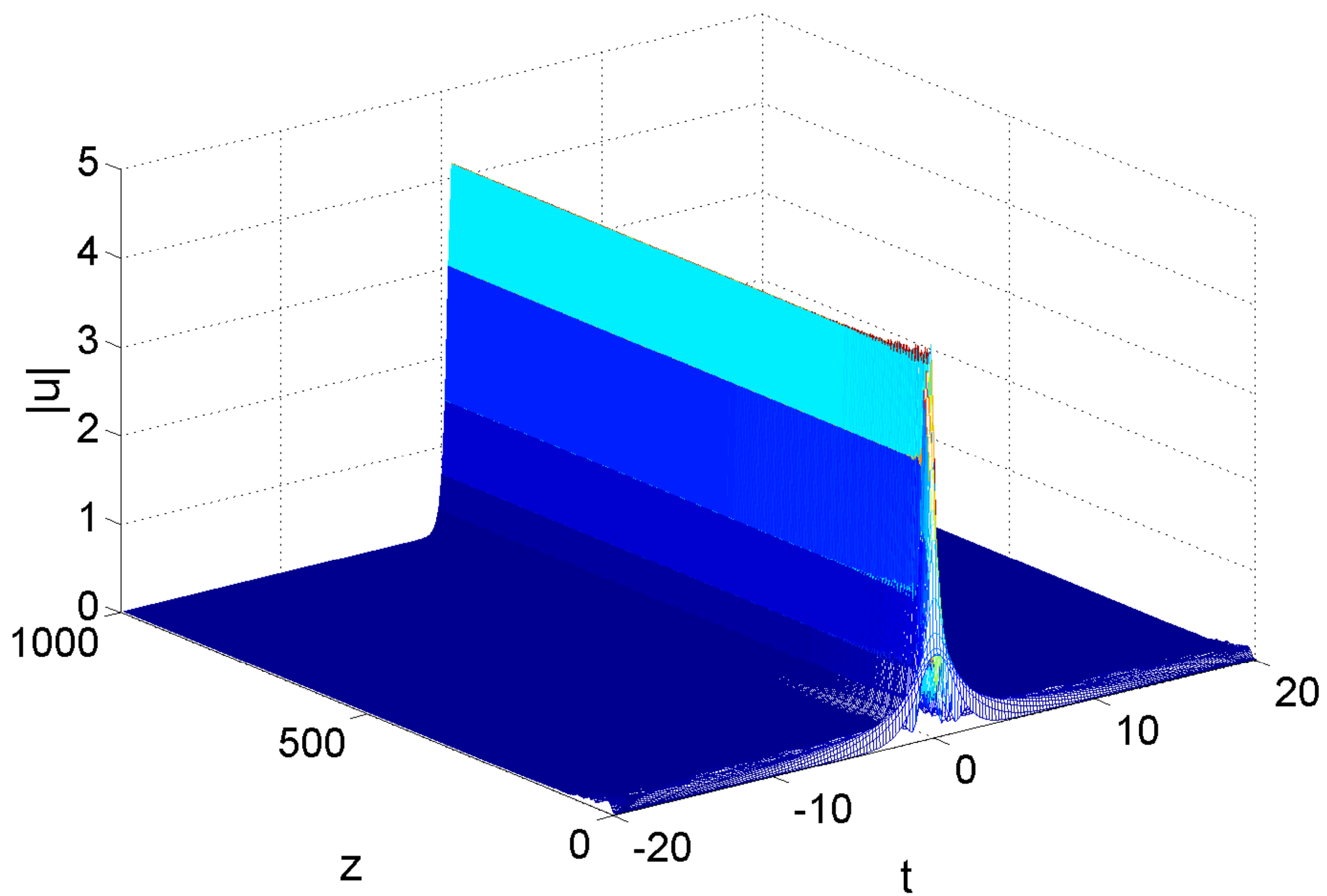
Stable pulse for $g = 0.3$



Evolution of the soliton peak for $g = 0.3$

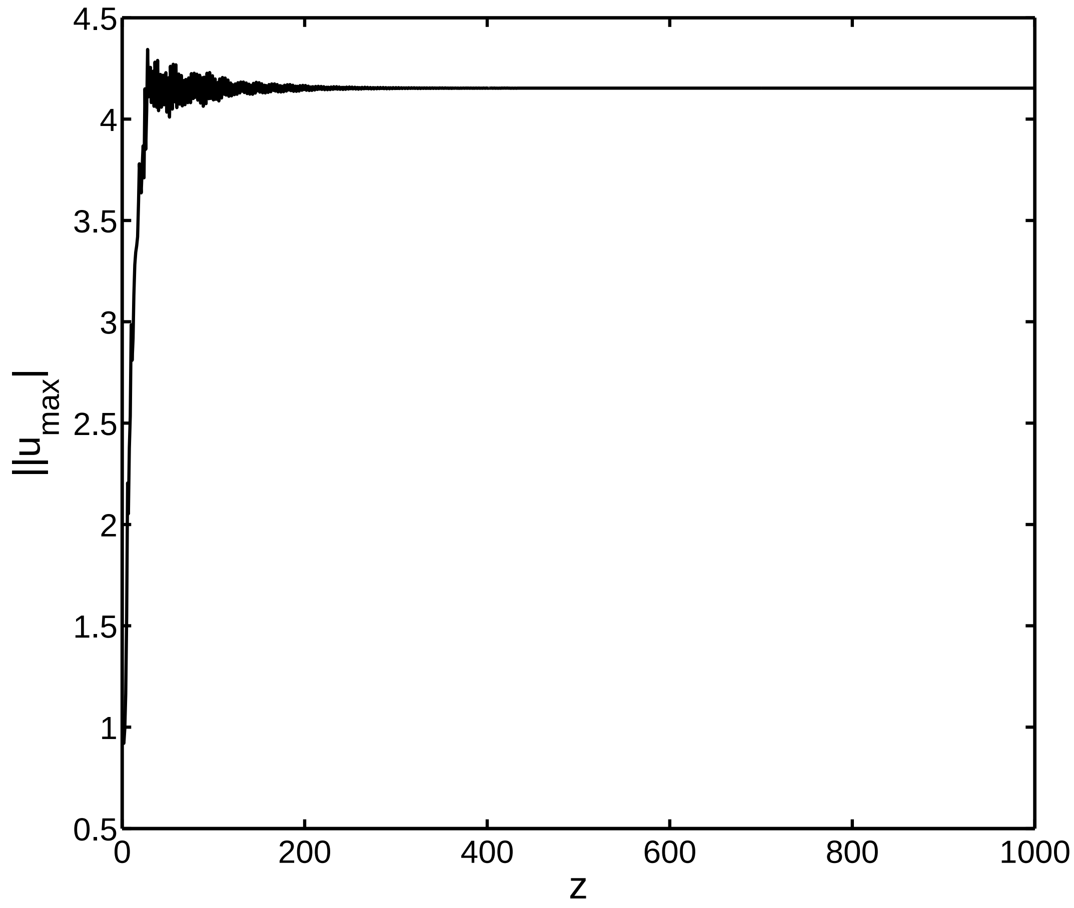


Stable pulse for $g = 0.7$



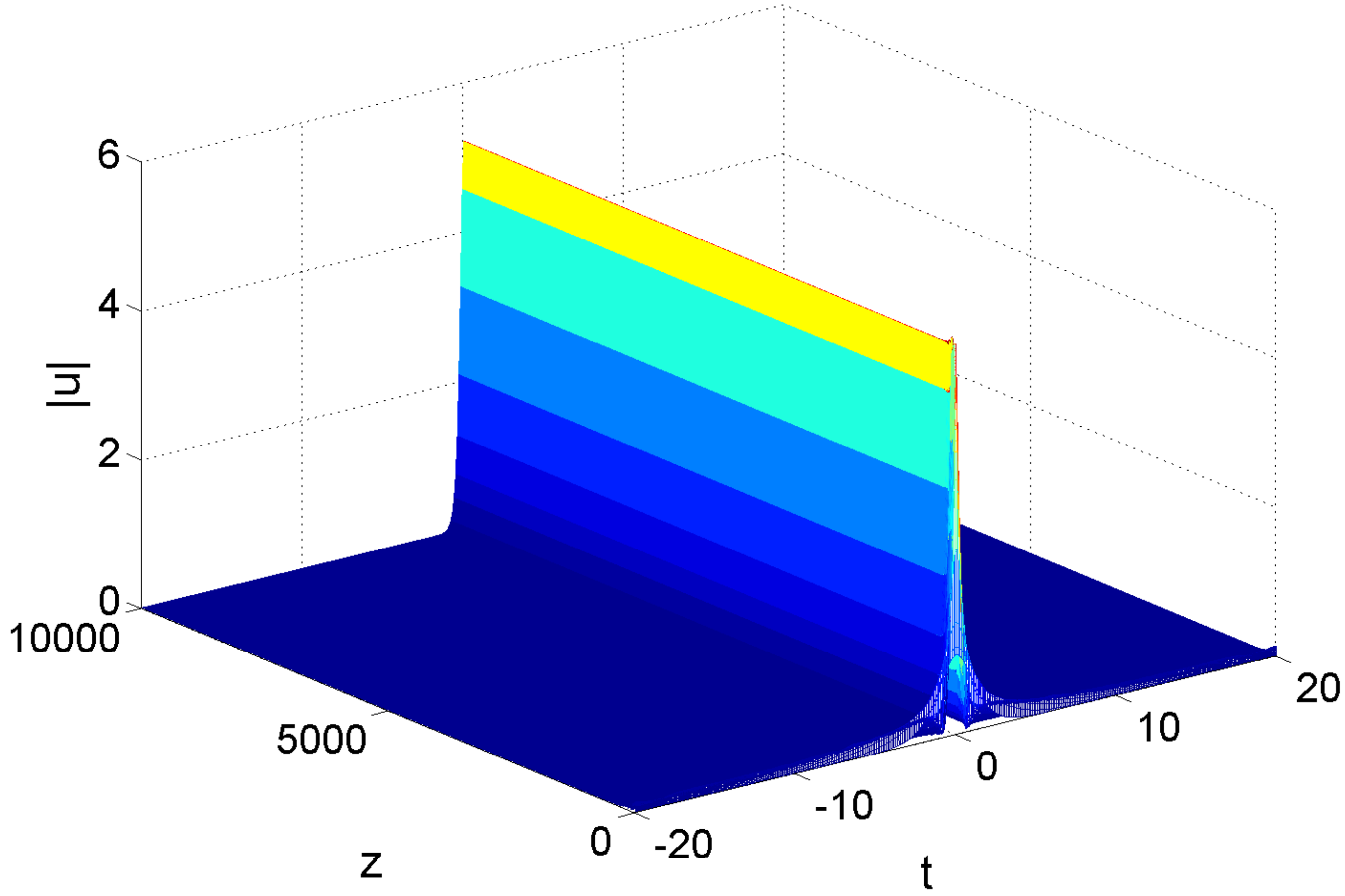
Dynamics of the mode–locking equation

Evolution of the soliton peak for $g = 0.7$

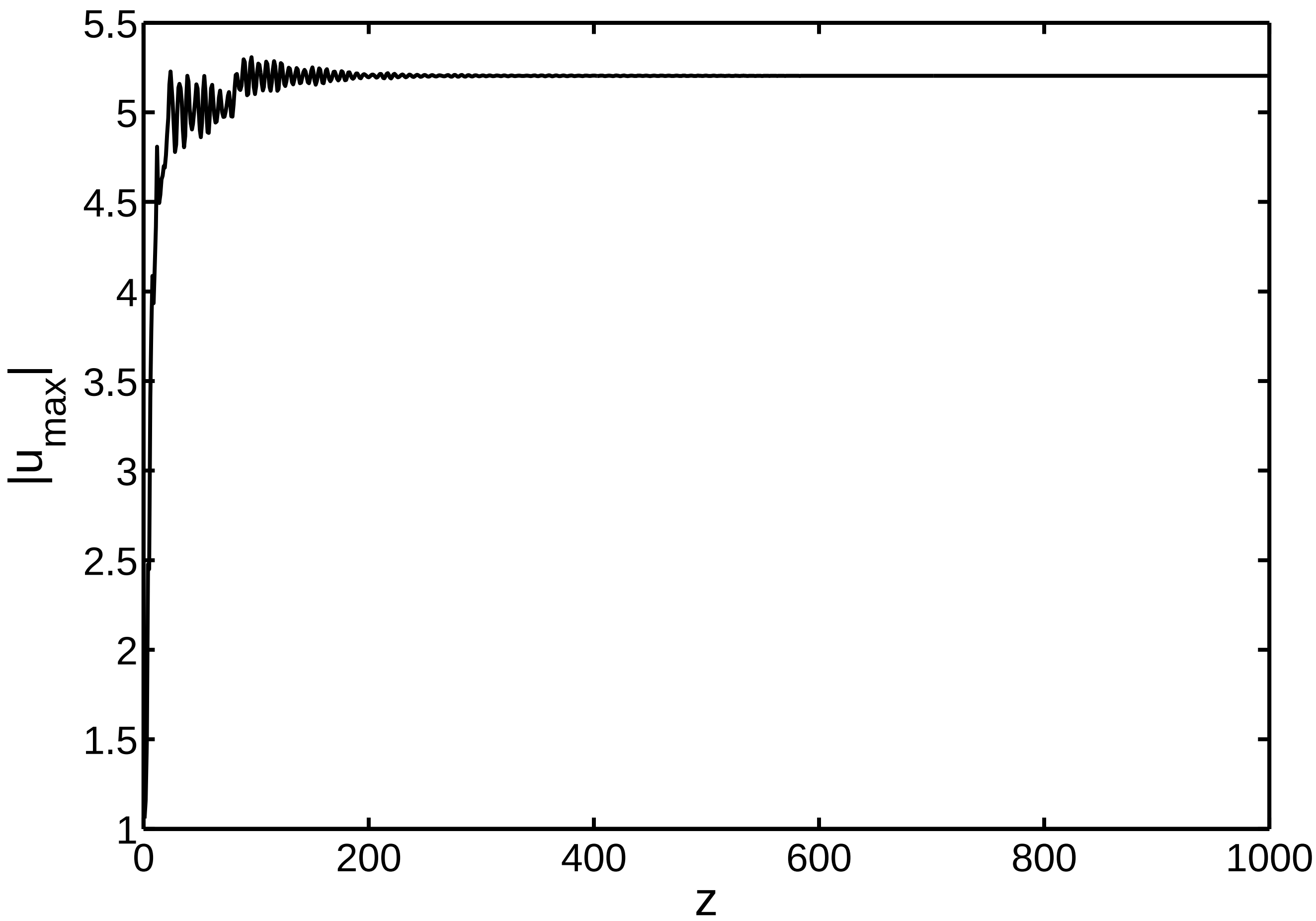


Dynamics of the mode–locking equation

Stable pulse for $g = 1$



Evolution of the soliton peak for $g = 1$



Denote

$$P(t) = \frac{ig}{1+\epsilon\int_{-\infty}^{+\infty}|u|^2\,\mathrm{d}t}u + \frac{i\tau}{1+\gamma\int_{-\infty}^{+\infty}|u|^2\,\mathrm{d}t}u_{tt} - \frac{il}{1+\delta|u|^2}u.$$

Using spectral renormalization we derive the following iteration scheme:

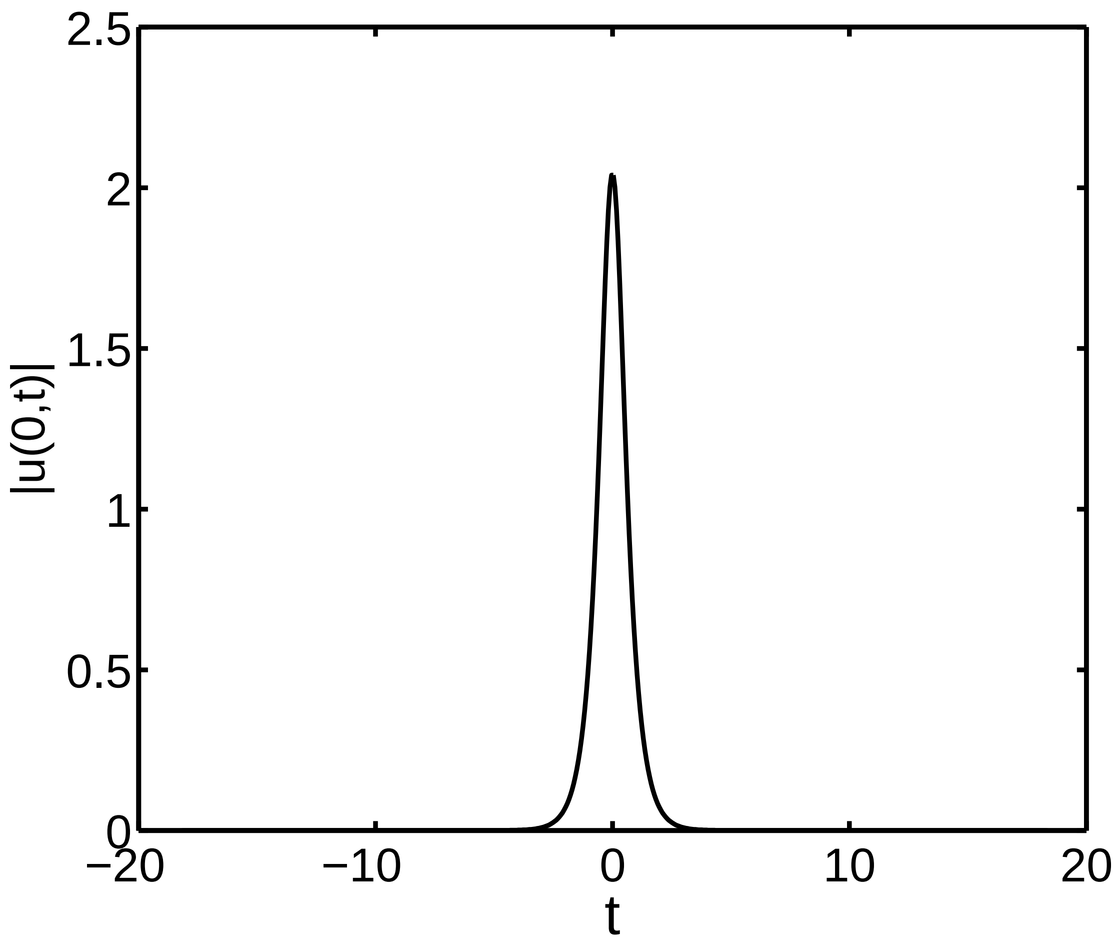
$$\int_{-\infty}^{+\infty}(\mu+\omega^2/2)|\hat{v_n}|^2\,\mathrm{d}\omega +$$

$$\int_{-\infty}^{+\infty}(\mathcal{F}\{P_n(\lambda,t)-|\lambda_n v_n|^2 v_n\})\hat{v_n}^*\,\mathrm{d}\omega = 0$$

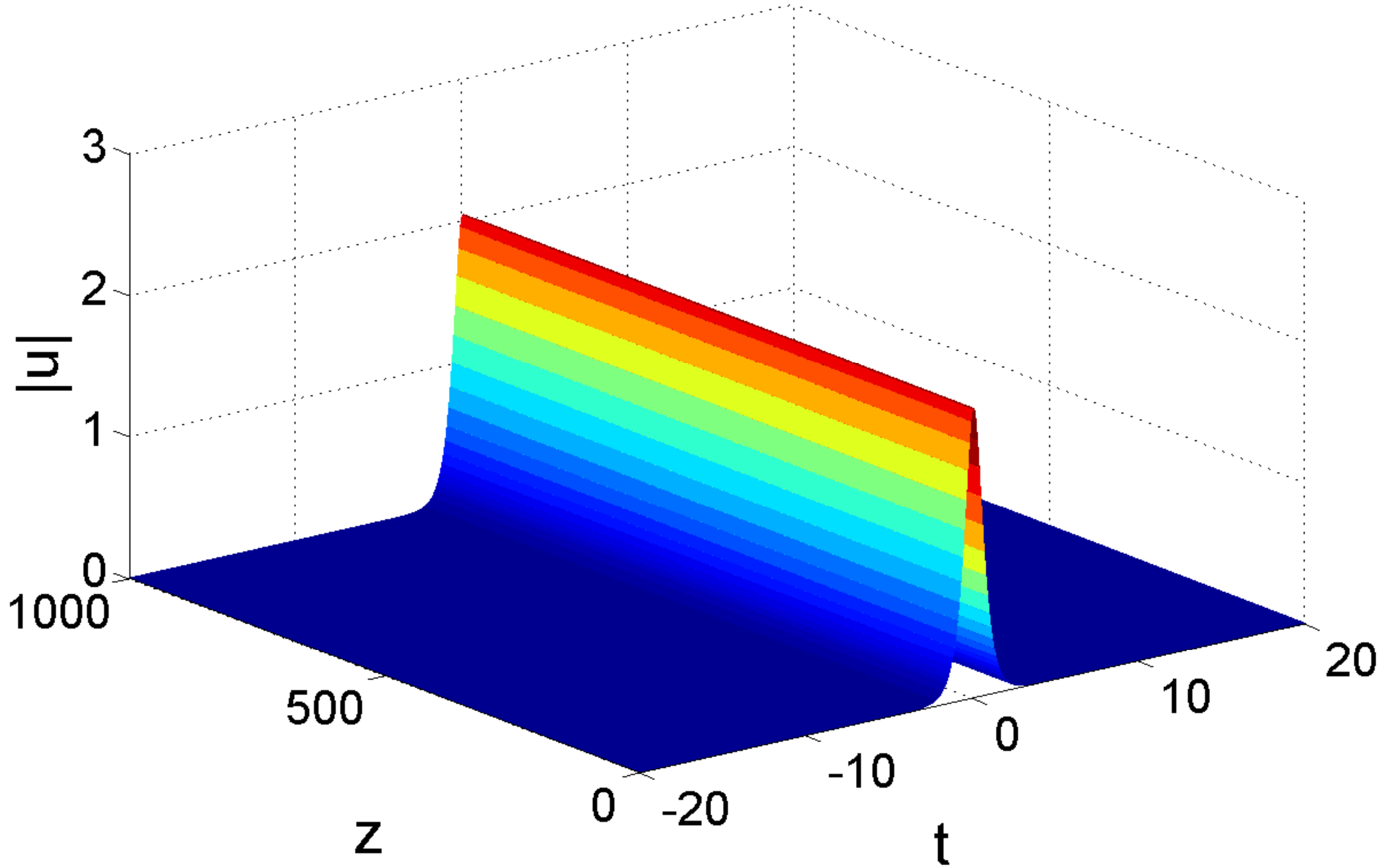
$$\hat{v}_{n+1} = -\frac{\mathcal{F}\{P_n(\lambda,t)-|\lambda_n v_n|^2 v_n\}}{\mu+\omega^2/2}$$

Again, modes, ie solutions to the equation are found for specific values of μ.

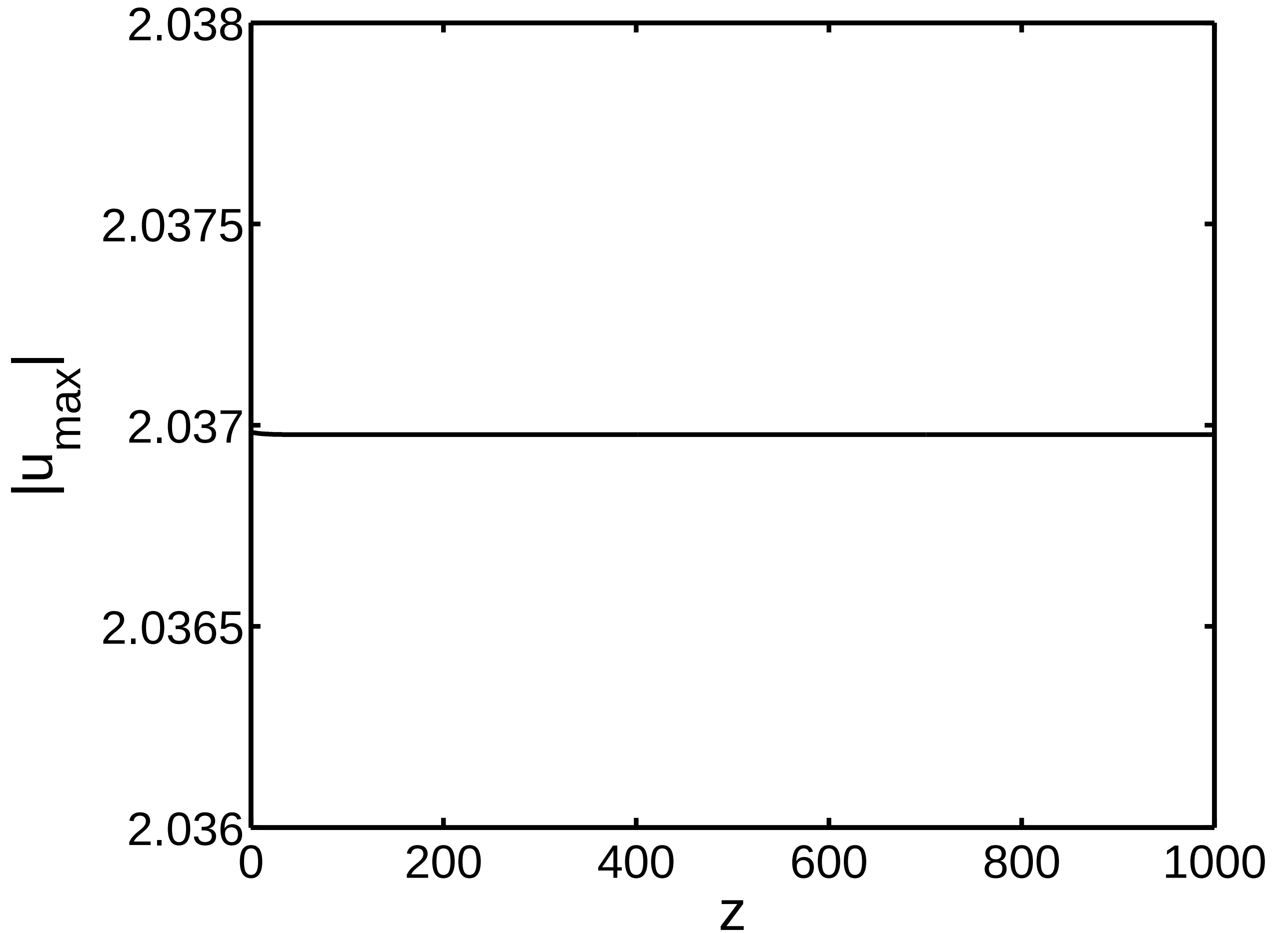
Stable pulse for $g = 0.3$ and $\mu = 2.09$



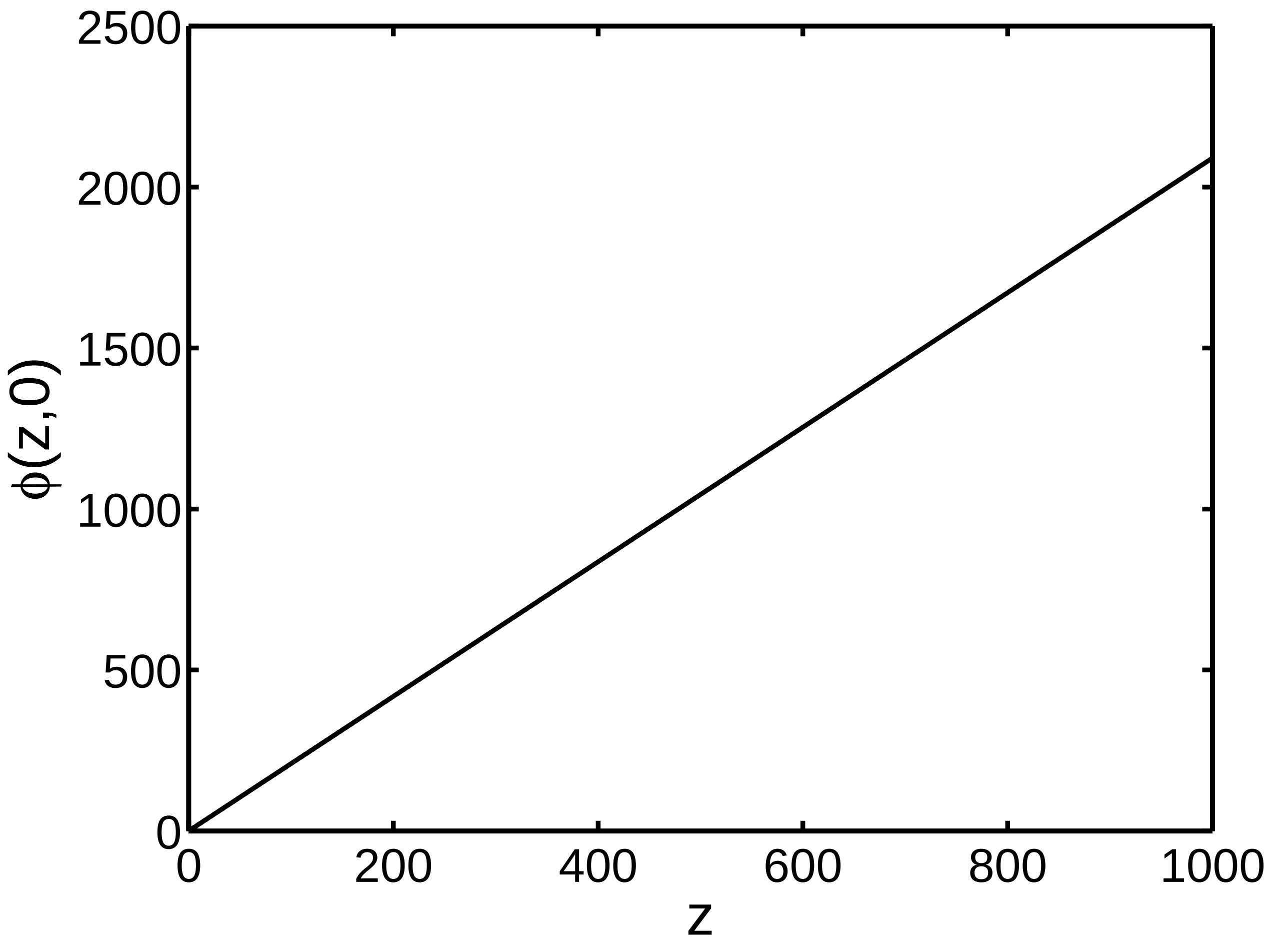
Evolution of the pulse for $g = 0.3$ and $\mu = 2.09$



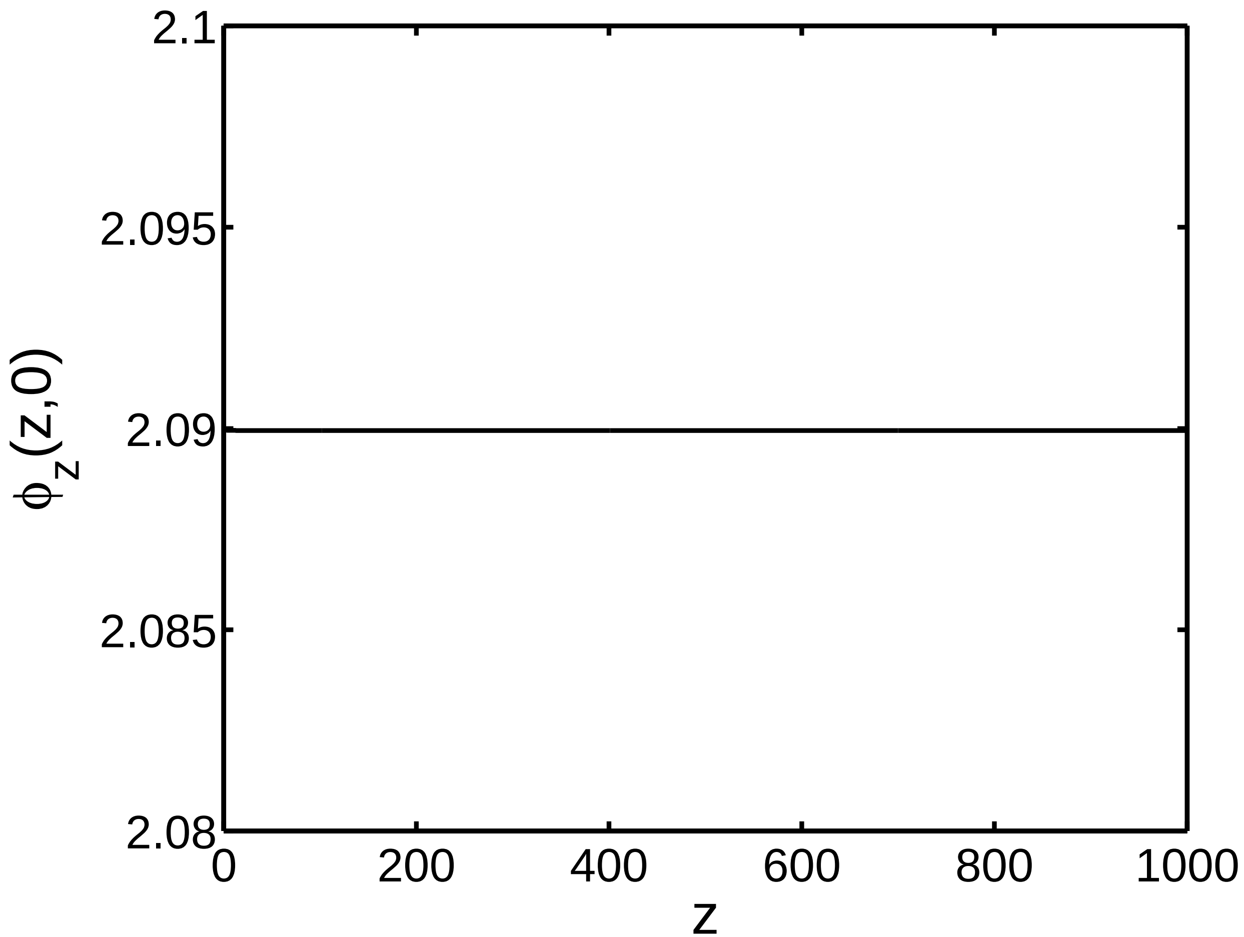
Evolution of the soliton peak for $g = 0.3$ and $\mu = 2.09$



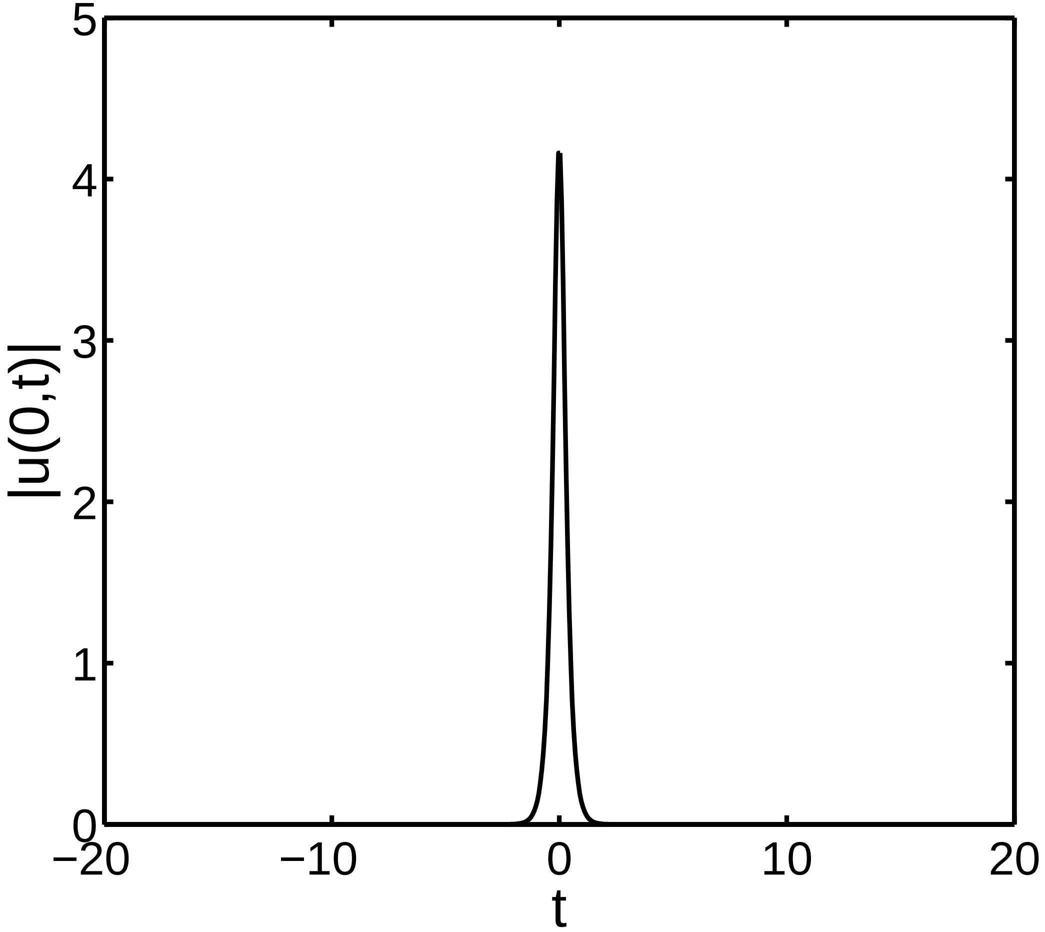
Evolution of the soliton phase $\psi(z)$ for $g = 0.3$ and $\mu = 2.09$



Evolution of $\phi_z (= \mu)$ for $g = 0.3$ and $\mu = 2.09$

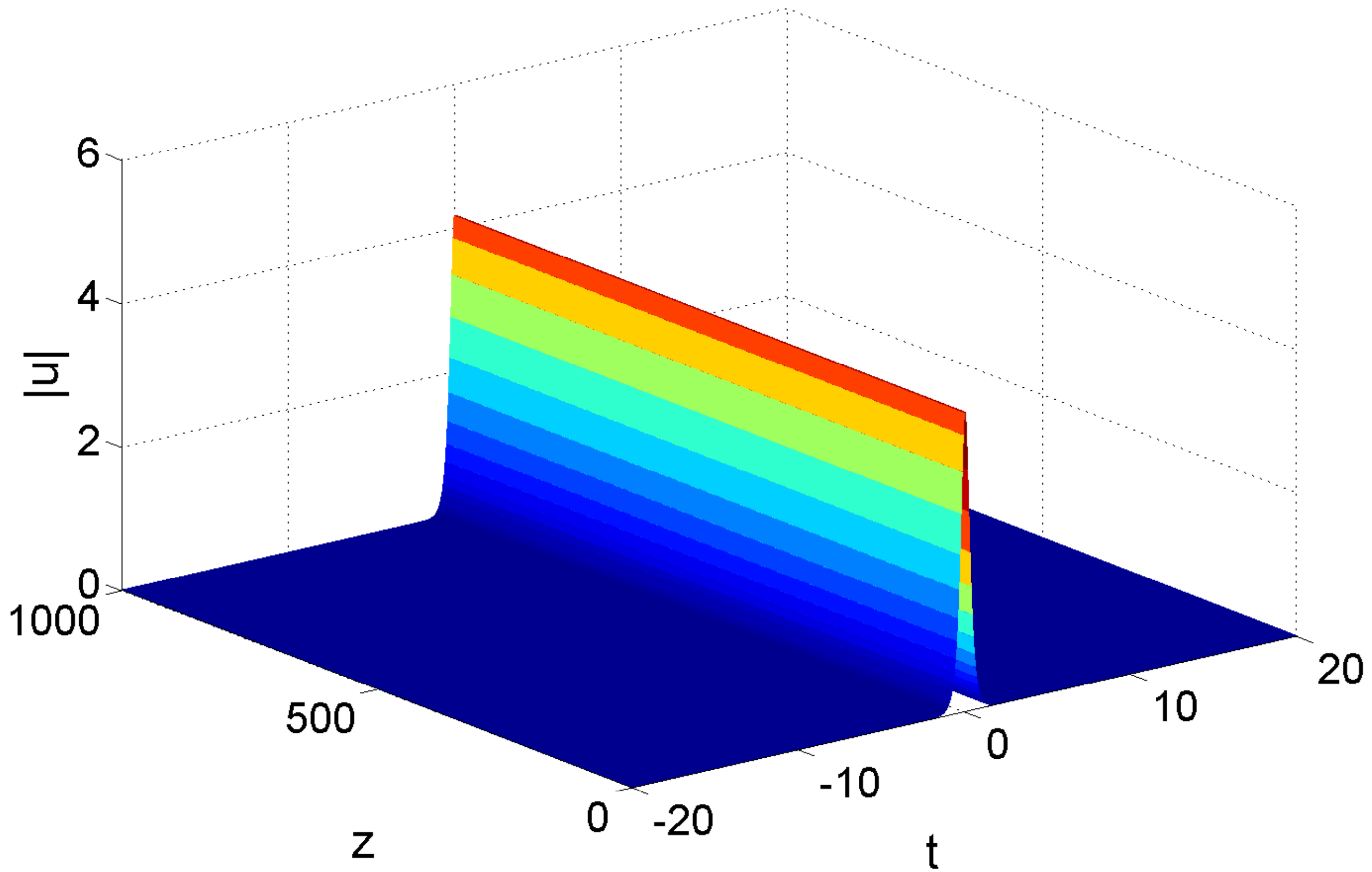


Stable pulse for $g = 0.7$ and $\mu = 8.83$



Solutions of the mode–locking equation

Evolution of the pulse for $g = 0.7$ and $\mu = 8.83$



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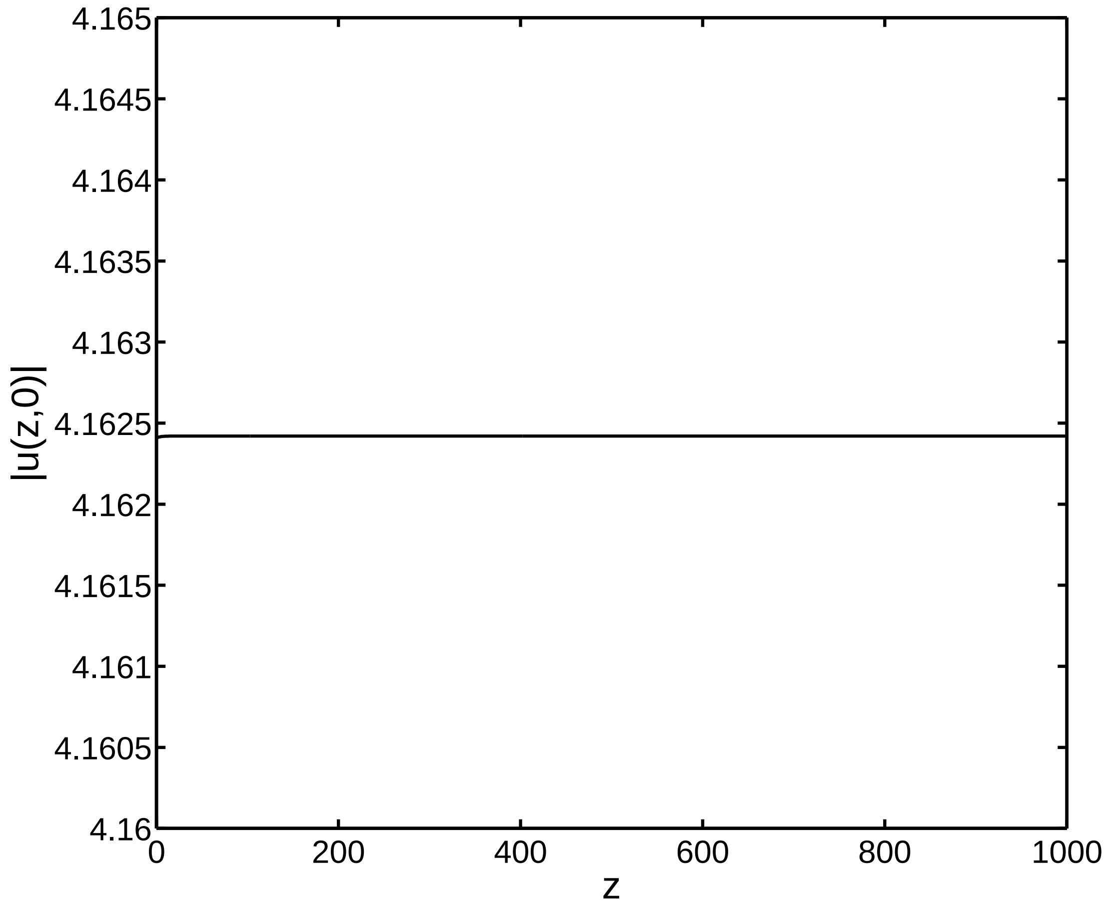
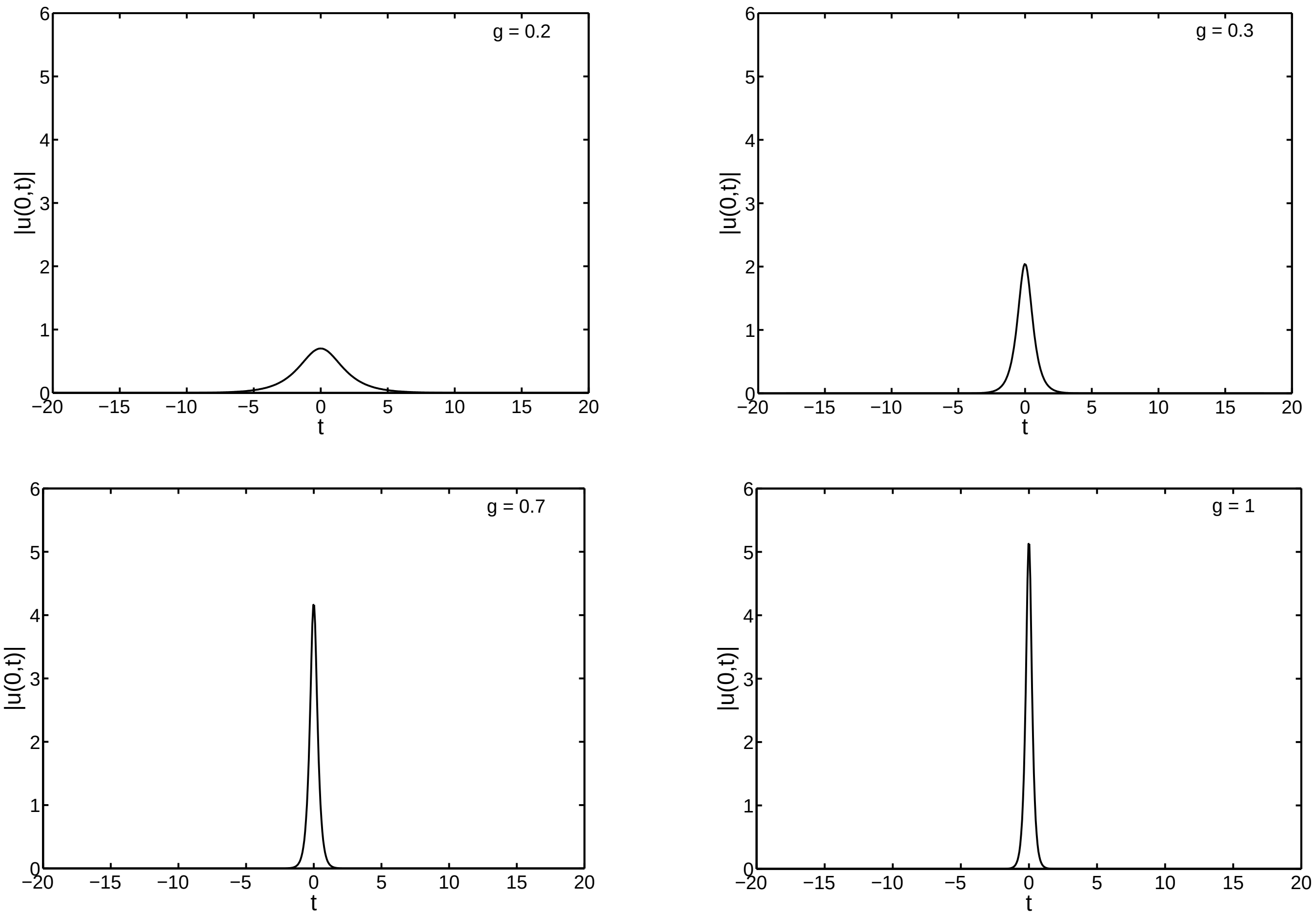


Solutions of the mode–locking equation

Solutions for different gain



Summary

- Mode–locking occurs for a wide range of parameters.

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- All modes are stable in their evolution.

Dispersion managed solitons

The characteristics of an optical pulse propagating in a dispersion managed (DM) system under the effect of gain and loss can be described by the p–NLS equation

$$i\frac{\partial u}{\partial z} + \frac{d(z)}{2}\frac{\partial^2 u}{\partial t^2} + g(z)|u|^2 u =$$
$$\frac{ig}{1+\epsilon\int_{-\infty}^{+\infty}|u|^2\,\mathrm{d}t}u + \frac{i\tau}{1+\gamma\int_{-\infty}^{+\infty}|u|^2\,\mathrm{d}t}u_{tt} - \frac{il}{1+\delta|u|^2}u$$

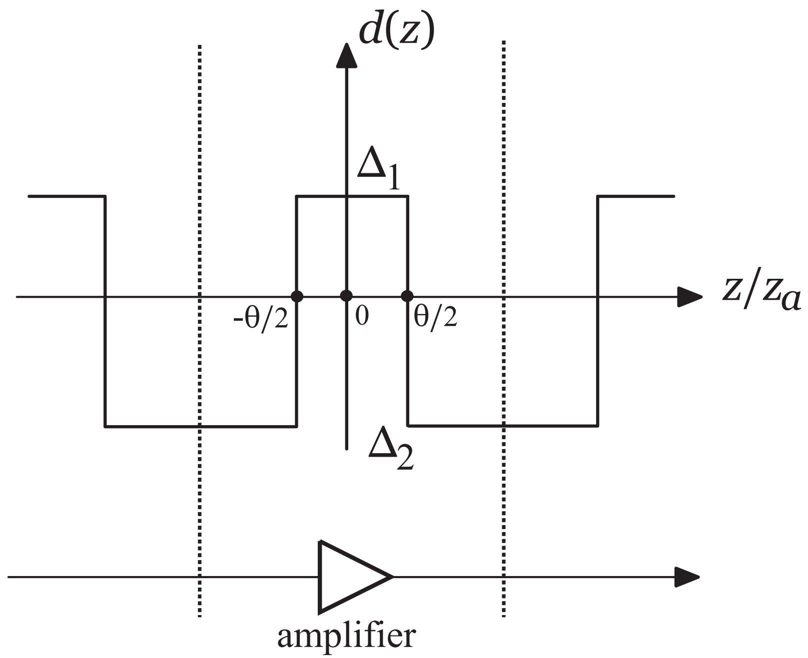
where $u = u(z,t)$, z, and t are the dimensionless variables which represent the complex envelope of the electrical field, longitudinal distance, and retarded time, respectively. The function $d(z)$ is the local value of fiber dispersion and $g(z)$ includes the effects of varying power resulting from fiber loss and periodic amplification. Both factors are real functions of z with period z_a, which denotes the DM period and which is taken equal to the amplifier spacing.

Dispersion maps

Define

$$d(z) = d_0 + \frac{\Delta(z/z_a)}{z_a}, \quad \langle \Delta \rangle = \int_0^1 \Delta(\zeta)\, \mathrm{d}\zeta = 0, \quad \zeta = \frac{z}{z_a}.$$

The parameters of a two-step map as illustrated as follows



DMNLS theory outline

- Define the new variables $\zeta = \dfrac{z}{z_a}$ (short scale), $Z = z$ (long scale).

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- Expand u in powers of $z_a \ll 1$.
- Substitute in the p–NLS equation and equate coefficients of equal powers of z_a.
- Solve the leading order equation of $O(z_a^{-1})$ and use the solution, $u^{(0)}$, to the $O(1)$ equation to derive the DMNLS equation.

DMNLS equation

The DMNLS equation has the form

$$i\frac{\partial \hat{U}_0}{\partial Z} - \frac{d_0}{2}\omega^2 \hat{U}_0 +$$

$$\int_0^1 \exp\left[i\frac{\omega^2}{2}C(\zeta)\right]\left(g(\zeta)\mathcal{F}\left\{|u^{(0)}|^2 u^{(0)}\right\} + \mathcal{F}\left\{P(u^{(0)})\right\}\right)\,\mathrm{d}\zeta = 0$$

where

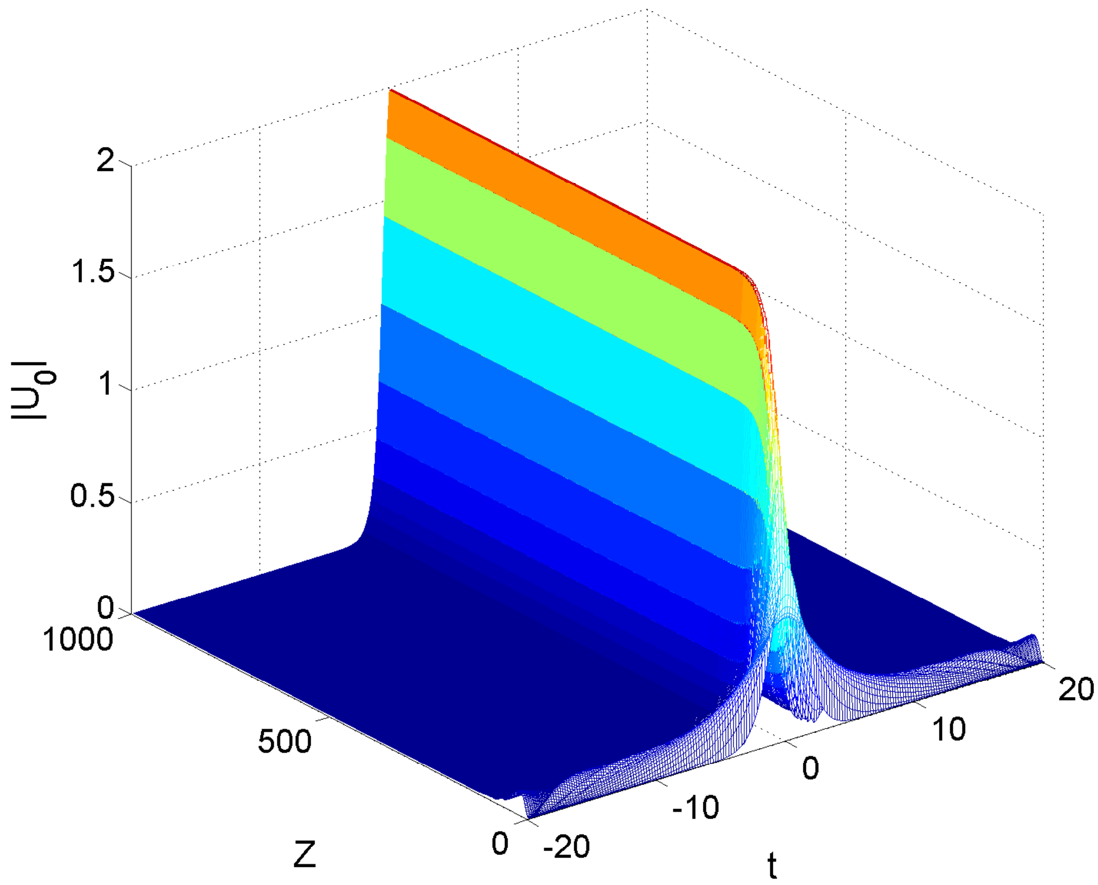
$$\hat{u}^{(0)}(\zeta, Z, \omega) = \exp\left[-i\frac{\omega^2}{2}C(\zeta)\right]\hat{U}_0(Z,\omega)$$

$$C(\zeta) = \int_0^\zeta \Delta(\zeta')\,\mathrm{d}\zeta'$$

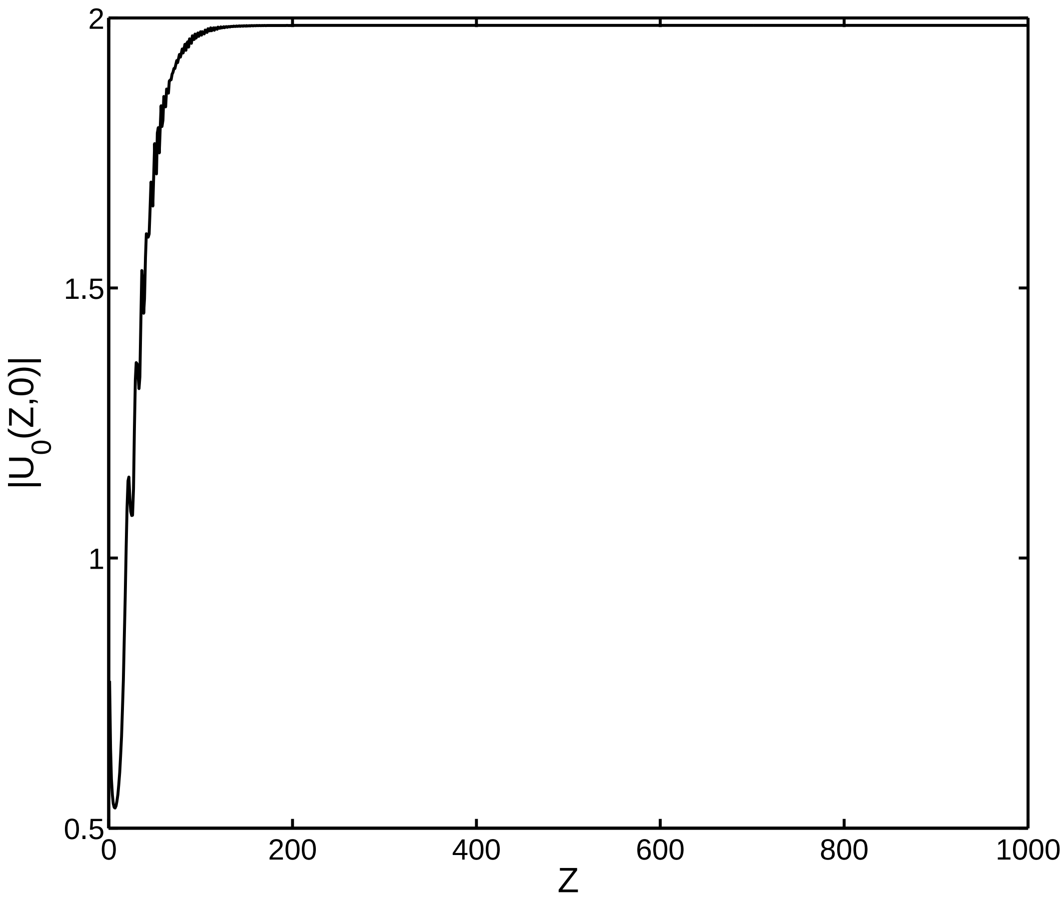
$$\hat{U}_0(Z,\omega) = \hat{u}^{(0)}(\zeta = 0, Z, \omega)$$

This is a partial differential equation for $\hat{U}_0(Z,\omega)$ and describes the long–scale dynamics of the pulse envelope.

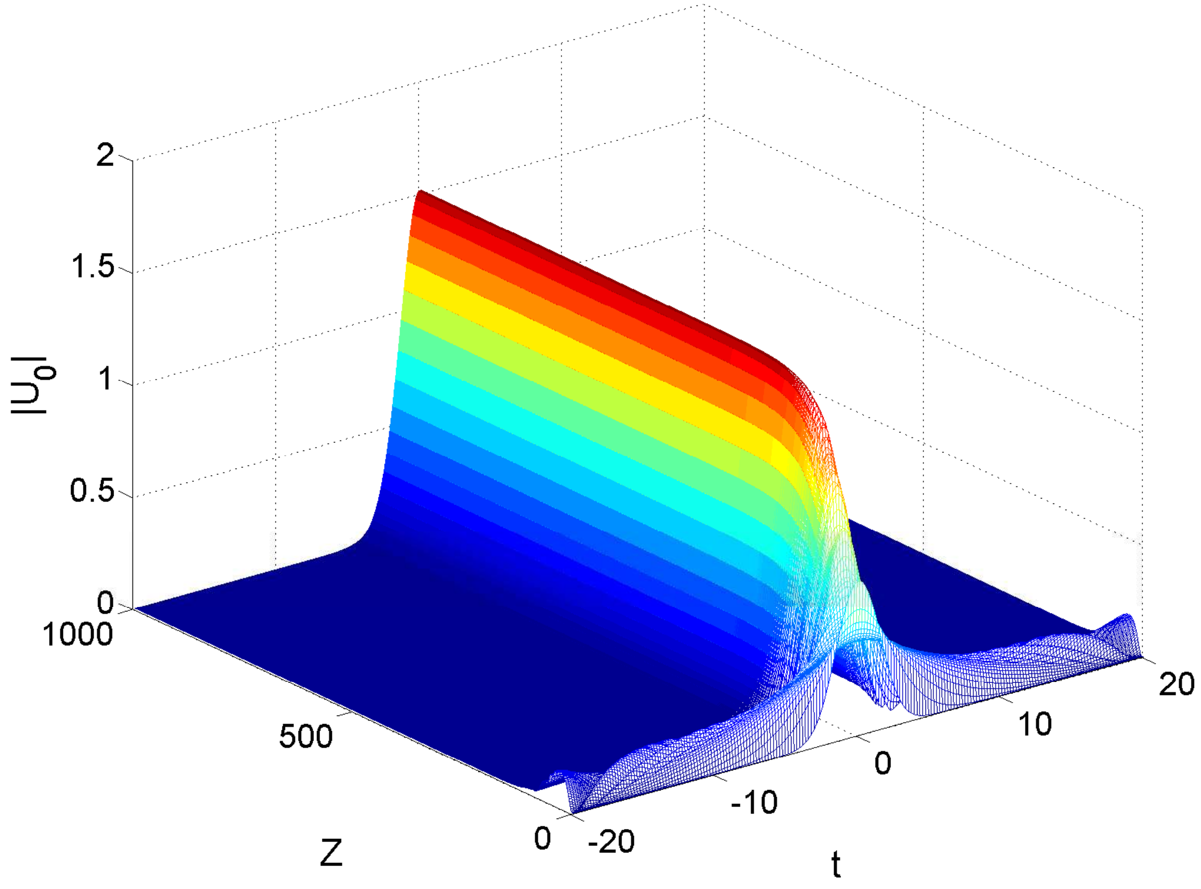
Weak dispersion management, $s = 0.1$, and $g = 0.3$



Evolution of the soliton peak for $s = 0.1$ and $g = 0.3$



Moderate dispersion management, $s = 1$, and $g = 0.3$



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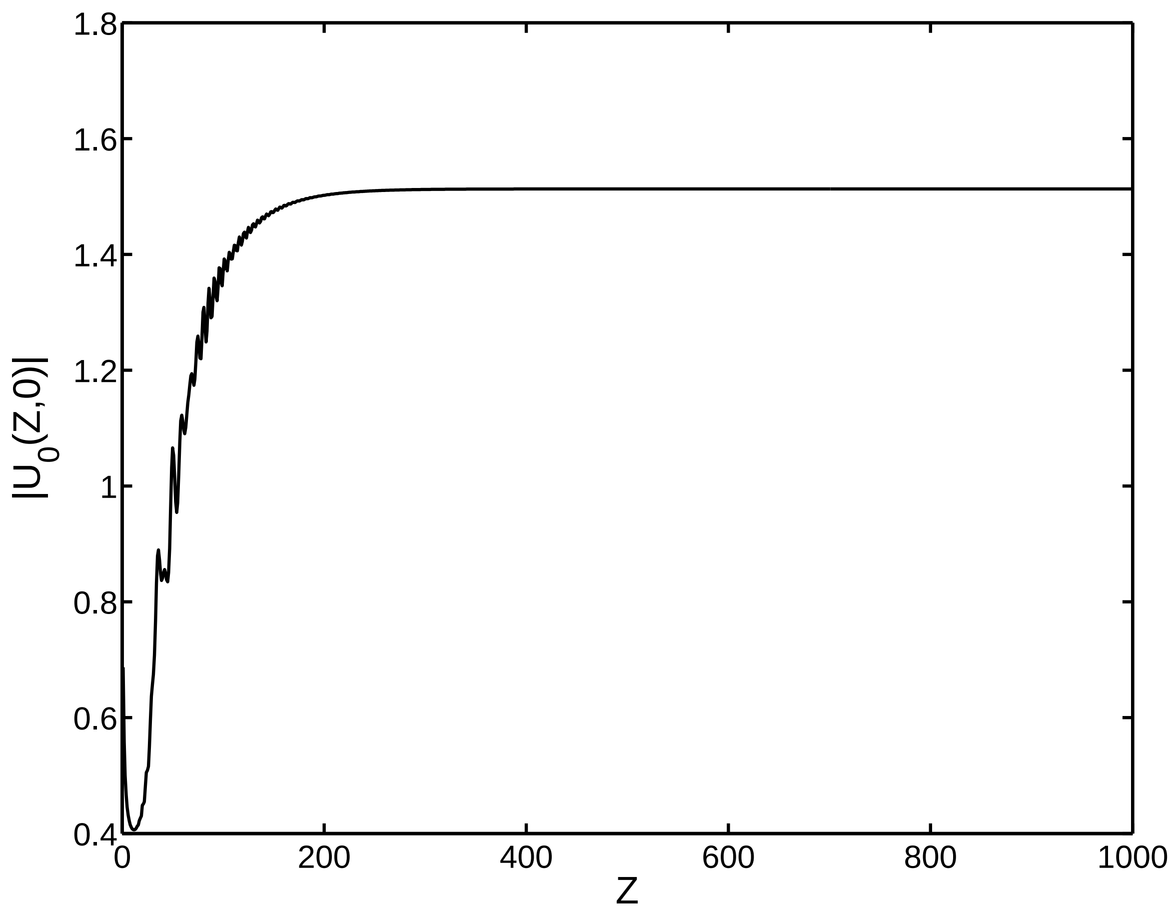
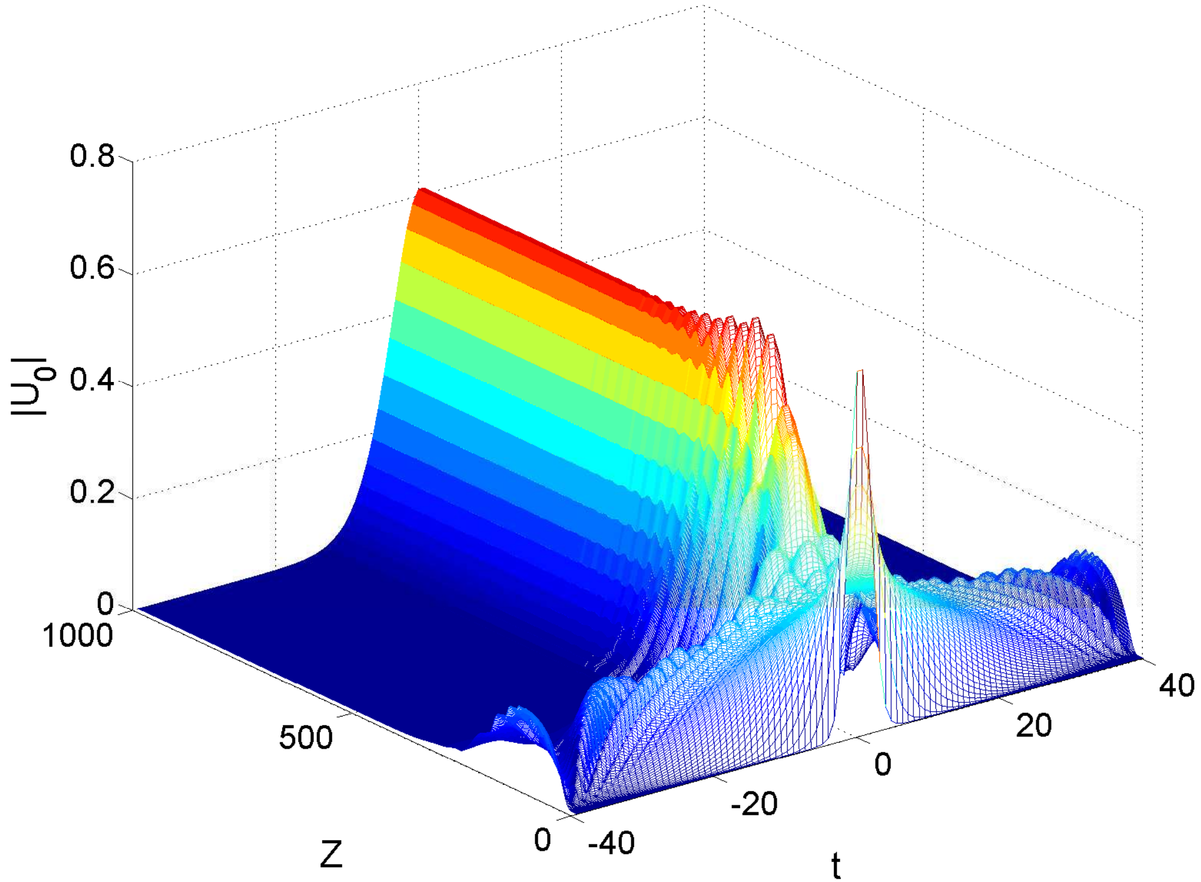
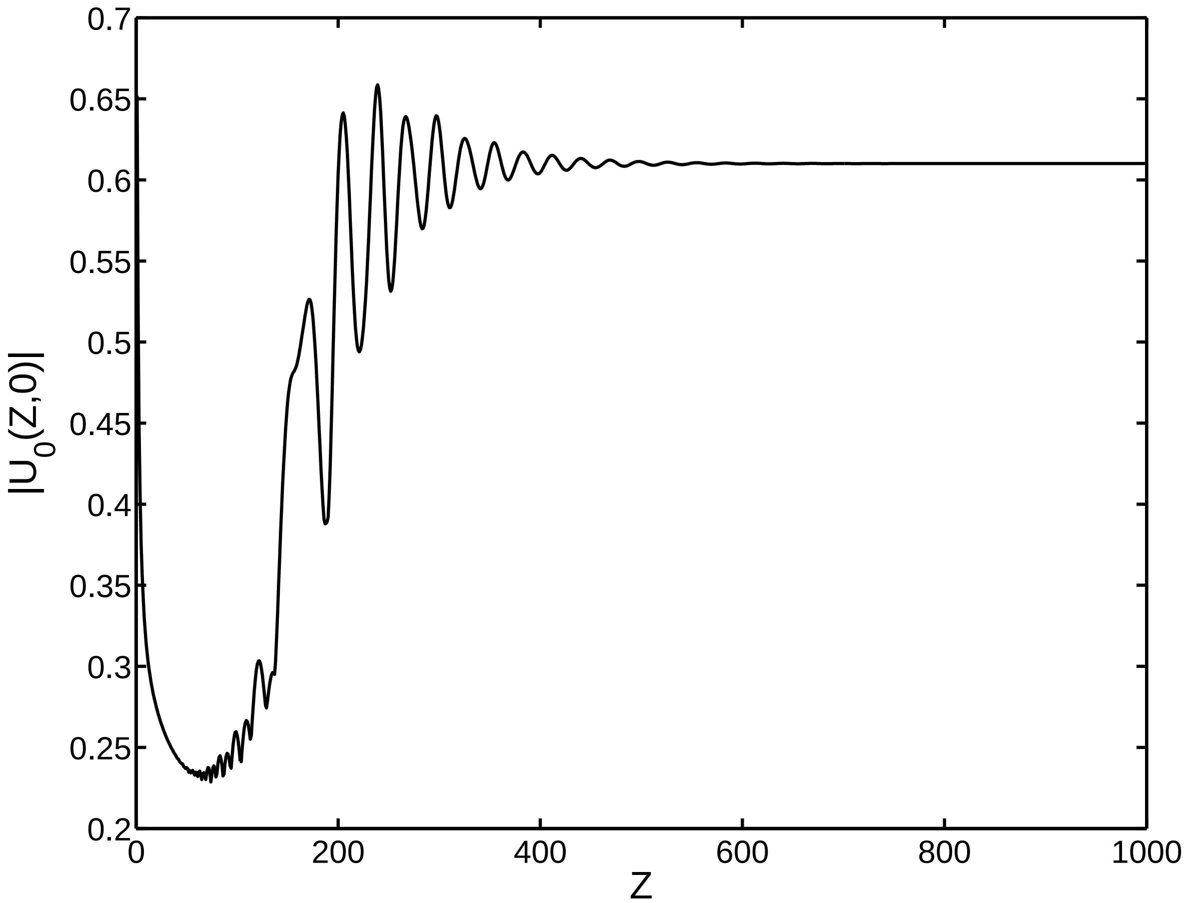


Strong dispersion management, $s = 10$, and $g = 0.3$

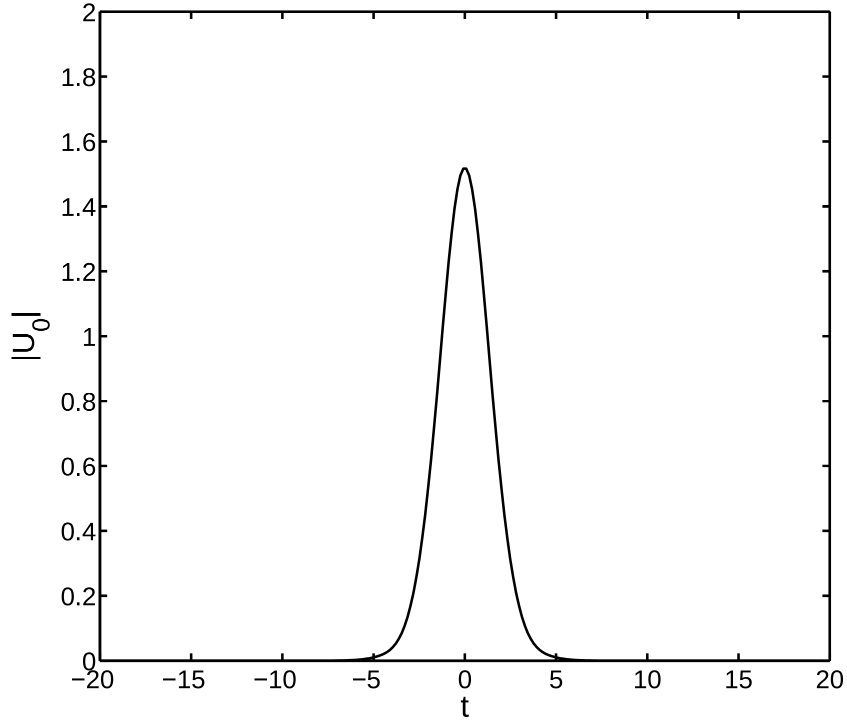


Evolution of the soliton peak for $s = 10$ and $g = 0.3$

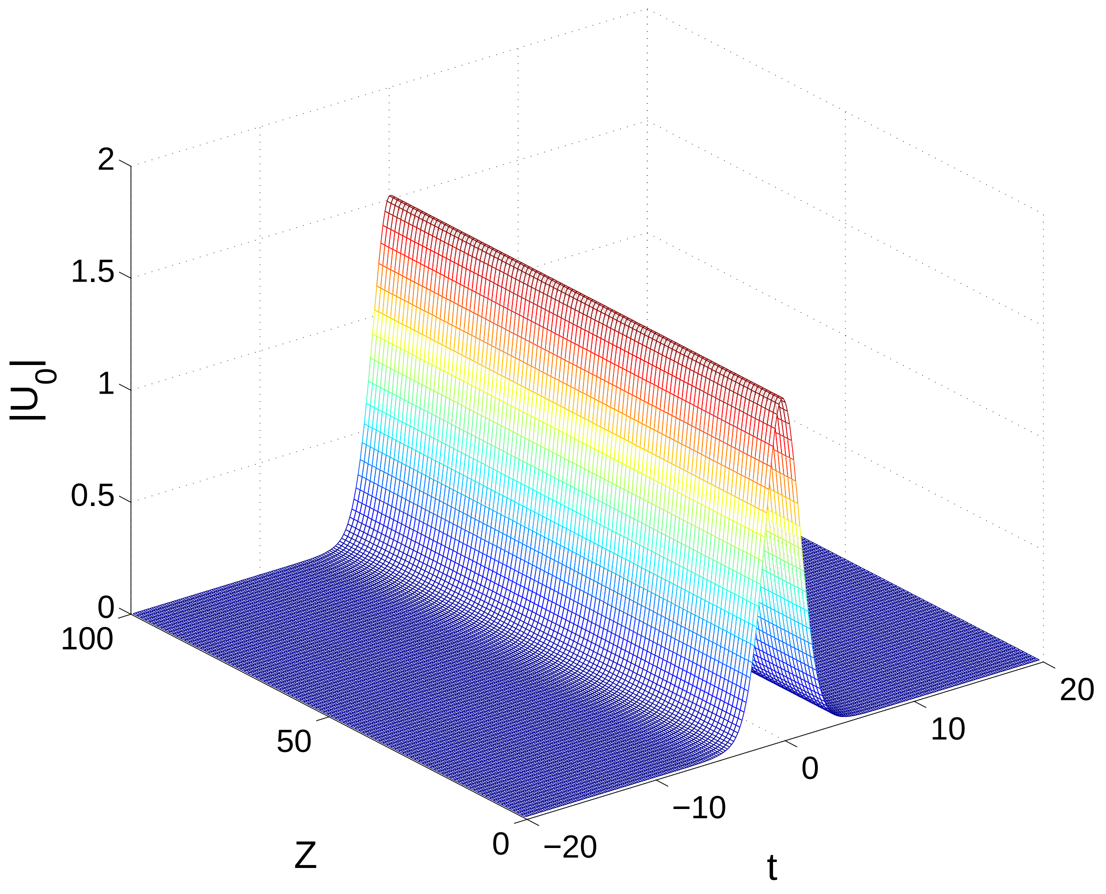


Solution of the DMNLS equation

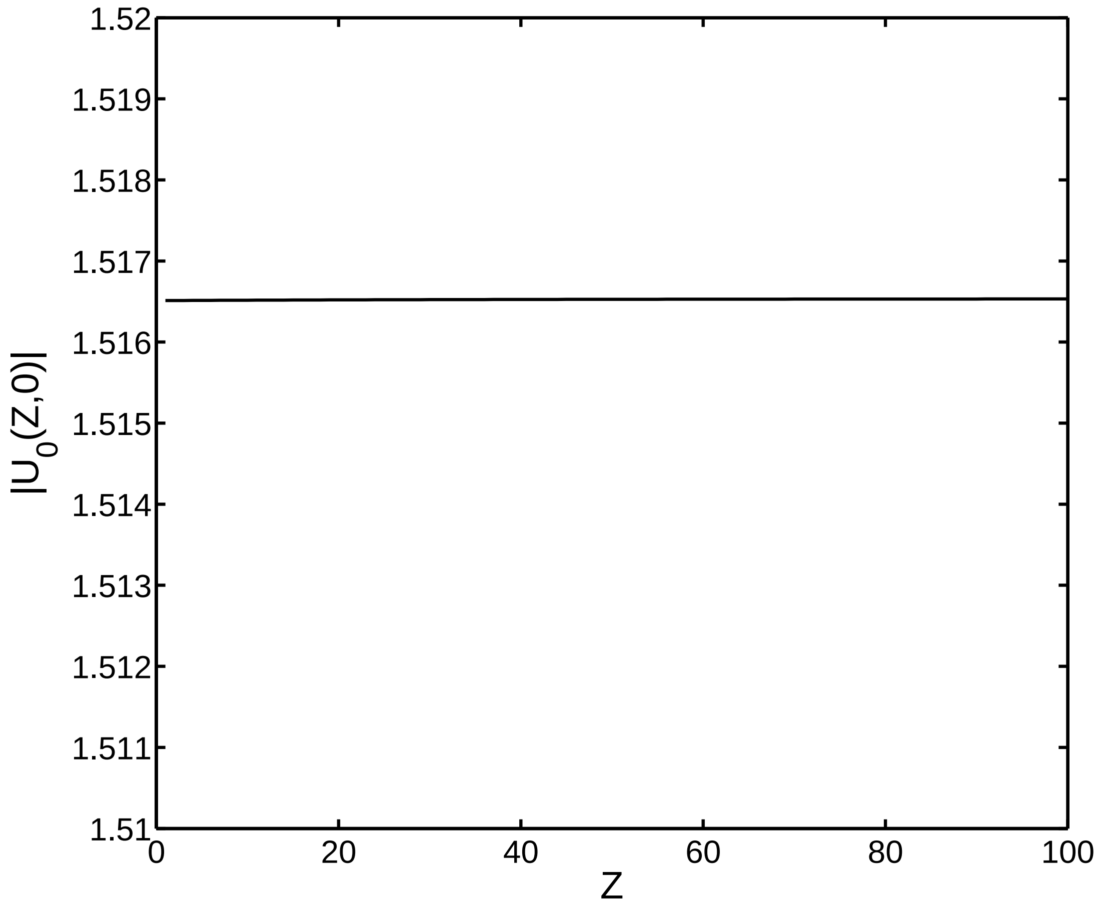
Moderate dispersion management, $s = 1$, $g = 0.3$ and $\mu = 1.7487$



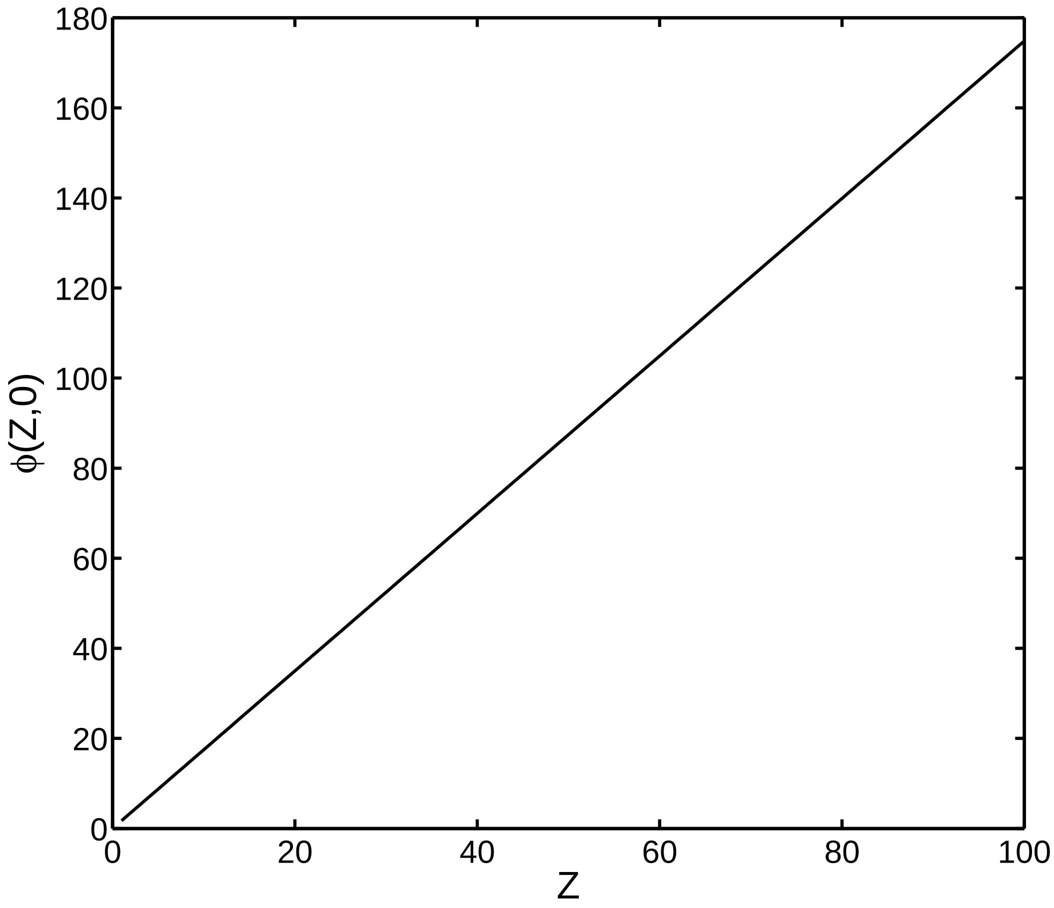
Evolution of the pulse for $s = 1$, $g = 0.3$ and $\mu = 1.7487$



Evolution of the soliton peak $s = 1$, $g = 0.3$ and $\mu = 1.7487$

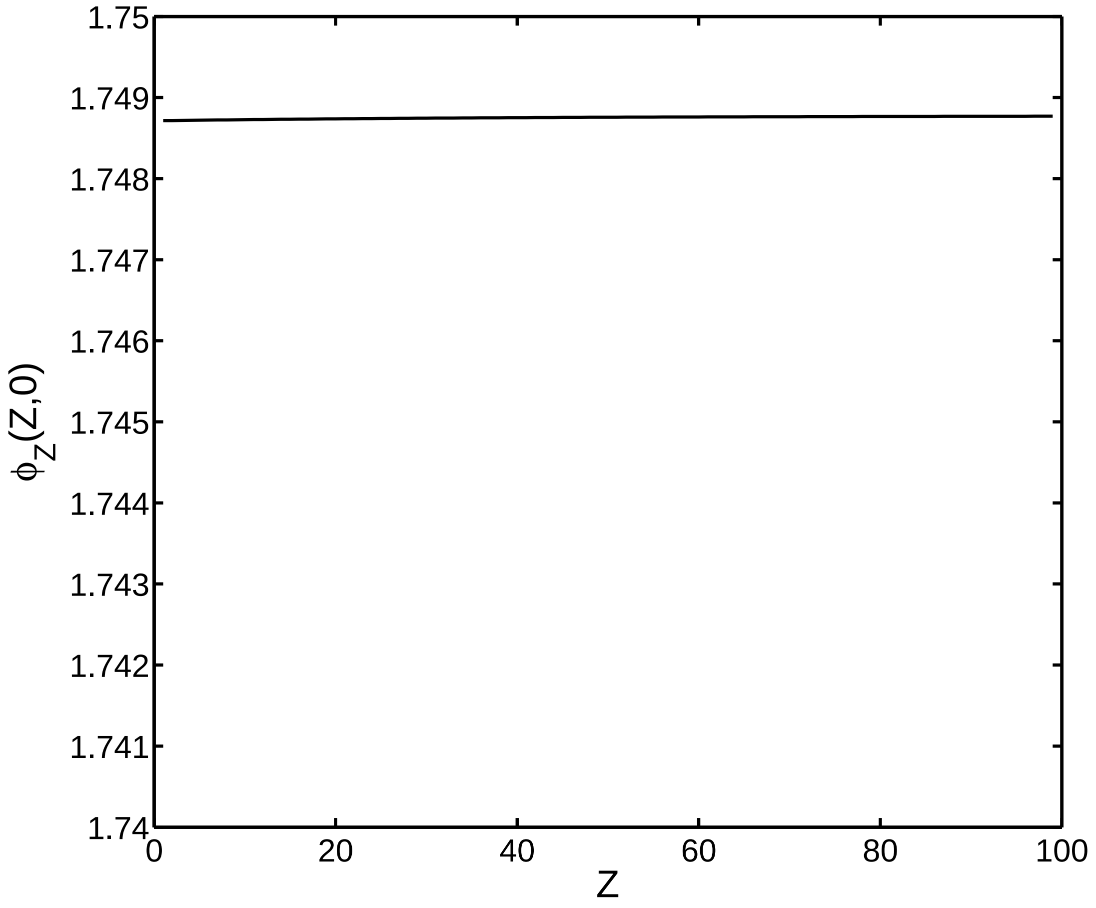


Evolution of the soliton phase $s = 1$, $g = 0.3$ and $\mu = 1.7487$



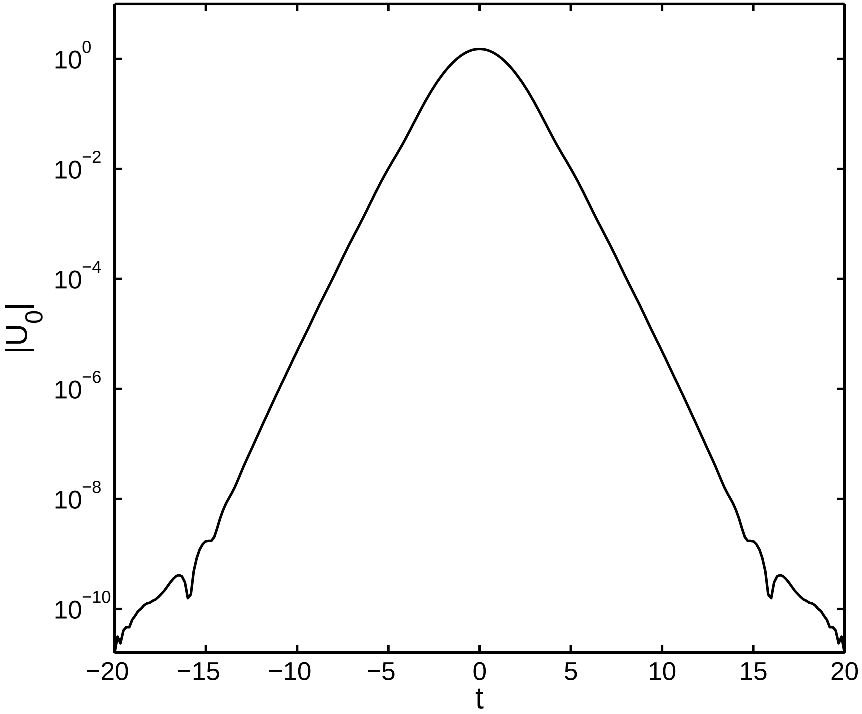
Solution of the DMNLS equation

Evolution of $\phi_Z(=\mu)$ for $s=1$, $g=0.3$ and $\mu=1.7487$



DMSNLS soliton in Log scale

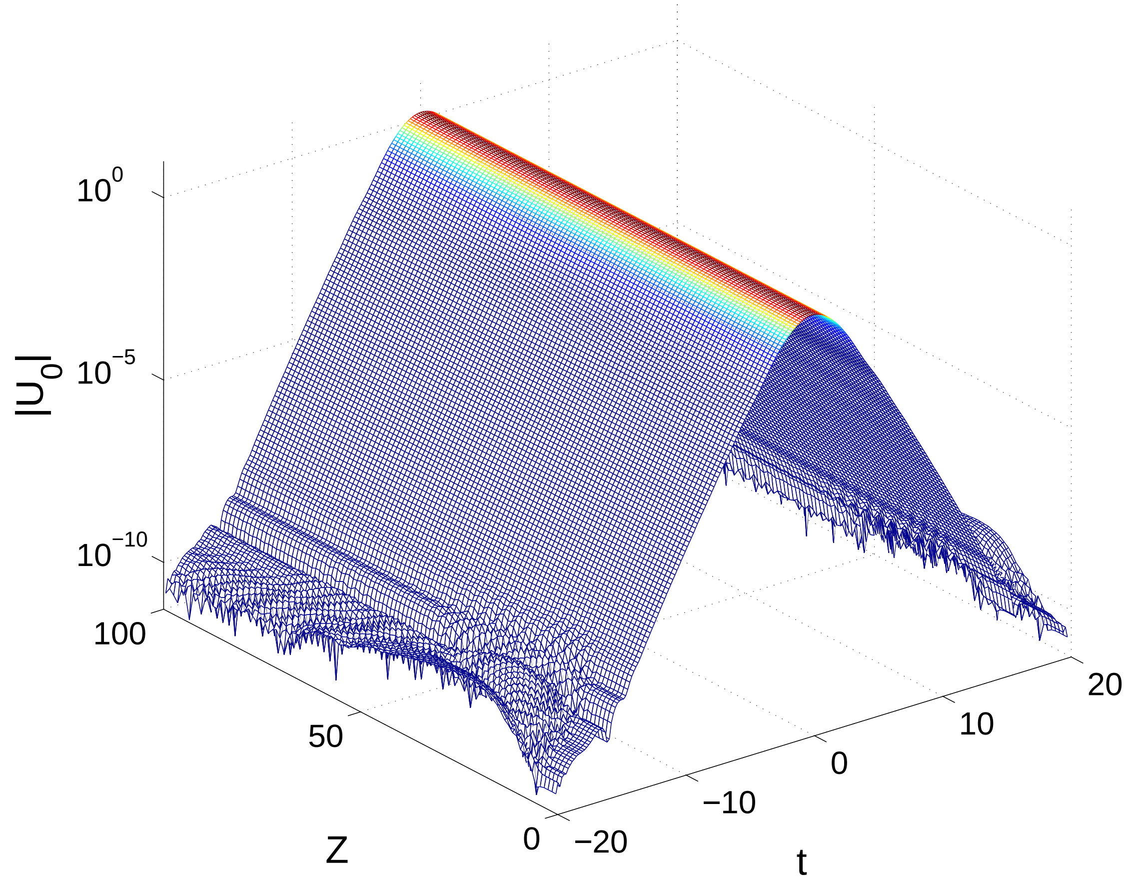
Moderate dispersion management, $s = 1$, $g = 0.3$ and $\mu = 1.7487$



DMSNLS soliton in Log scale

- Contents
- Mode–locked lasers
- Ti:Sapphire laser
- Spectral Renormalization
- Examples: Vanishing boundary conditions
- Nonvanishing boundary conditions
- The master equation
- The mode–locking equation
- **Dispersion managed solitons**
- Conclusions

Evolution of the pulse for $s = 1$, $g = 0.3$ and $\mu = 1.7487$



Summary

- DMNLS equation is an asymptotic equation that reassembles the features of the constant dispersion NLS.

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- DMNLS equation is an asymptotic equation that reassembles the features of the constant dispersion NLS.
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- The amplitude also increases with g.

Conclusions

We presented recent developments in the theory of mode–locked lasers. A new model equation is proposed for the study of these systems, that seems to be able to describe all their physical properties. A numerical method, the spectral renormalization, is used to find the solutions of this equation for both constant dispersion and dispersion managed solitons.